A Statistical Analysis on Modeling Uncertainty Through Crack Initiation Tests

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ABSTRACT

Because a large time spread in most crack initiation tests makes it a daunting task to predict the initiation time of cracking, a probabilistic model, such as the Weibull distribution, has been usually employed to model it. In this case, although it might be anticipated to develop a more reliable cracking model under ideal cracking test conditions (e.g., large number of specimen, narrow censoring interval, etc.), it is not straightforward to quantitatively assess the effects of these experimental conditions on model estimation uncertainty. Therefore, we studied the effects of some key experimental conditions on estimation uncertainties of the Weibull parameters through the Monte Carlo simulations. Simulation results suggested that the estimated scale parameter would be more reliable than the estimated shape parameter from the tests. It was also shown that increasing the number of specimen would be more efficient to reduce the uncertainty of estimators than the more frequent censoring.

1. INTRODUCTION

It is widely known that stress corrosion cracking (SCC) can result in loss-of-coolant accidents in nuclear reactors (Scott and Meunier 2007, Lunceford, DeWees et al. 2013, Kim and Do 2015). Thus, the prediction of the SCC initiation time is a very important task for several researchers in nuclear science. However, this is a difficult task due to the complex mechanism of SCC initiation, which is not clearly identified yet. Therefore, empirical SCC initiation models are generally adopted for this purpose (Amzallag, Hong et al. 1999, Garud 2009, Erickson, Ammirato et al. 2011).

However, most SCC experiments showed non-negligible scatter with respect to cracking time (Troyer, Fyfitch et al. August 9-13, 2015), although all of the experimental conditions (e.g., temperature, tensile stress, etc.) were strictly controlled. Therefore, a probabilistic model was frequently used as an SCC initiation model to quantitatively consider the time scatter. Particularly, the Weibull distribution (Weibull 1951), which can generally consider the effect of the time-dependent degradation of a material, is widely accepted as a probabilistic model of SCC initiation time (Hwang, Kwon et al. 2001, Eason 2005, Erickson, Ammirato et al. 2011).

To obtain the model parameters of SCC initiation (i.e., Weibull parameters in this case), a cracking test must be performed. The typical procedure of a cracking test involves an interval-censored reliability test. This implies that several stressed specimens (e.g., U-bend or constant tensile stress specimens) are exposed to a corrosive environment and censored at every scheduled time. Following the test, the testing results can be used to estimate the Weibull parameters typically using the maximum likelihood estimation (MLE) (McCool 2012). It is expected that the reliability of the estimated Weibull parameters would increase with an increase in the number of test specimens and a smaller length of the censoring interval (LCI). However, there is no theory yet available to calculate the exact estimation uncertainties for the estimated Weibull parameters with interval-censored data (McCool 2012). Therefore, in this study, the effects of some key experimental conditions on estimation uncertainties of Weibull parameters were investigated through the Monte Carlo simulation.

2. SIMULATION APPROACH

The cumulative distribution function (CDF) of the two-parameter Weibull distribution is frequently used as a cracking probability function and given by:

$$F(t;\beta,\eta) = 1 - exp\left[-\left(\frac{t}{\eta}\right)^{\beta}\right], \qquad (1)$$

where $t \geq 0$ denotes time, $\beta > 0$ denotes the shape parameter and $\eta > 0$ denotes the scale parameter of the Weibull distribution.

As previously mentioned, maximum likelihood estimation (MLE) is usually used to estimate the Weibull parameters from the given test data for its good estimation efficiency (Genschel and Meeker 2010, Park and Bahn 2016). The likelihood function for the interval-censored case is given by:

$$L(\beta,\eta) = \prod_{i=1}^{S}[1-F(s_i;\beta,\eta)]\cdot\prod_{j=1}^{C}\left[F\left(c_{j_U};\beta,\eta\right)-F\left(c_{j_L};\beta,\eta\right)\right] \quad (2)$$

where S is the number of suspended specimens, s_i is the last censoring time of i_{th} suspended specimen, C is the number of interval-censored cracked specimens, and c_{j_U} and c_{j_L} are the upper and lower bound times, respectively, of the censoring interval for the j_{th} cracking. The sum of S and C is equal to the total number of specimens N. We used a numerical approach for MLE. In this case, the MATLAB (MathWorks, Ver. R2015b) offers the numerical nonlinear simultaneous equation solver *fsolve*.

Experimental factors (e.g., number of specimens) can affect the uncertainties of Weibull estimators. The experimental factors considered in the simulation study include (1) true Weibull parameters; (2) the number of specimens; (3) end cracking fractions (ECF); and (4) length of censoring interval (LCI). Table 1 shows the simulation range of the study. A total of 900(=1×3×10×3×10) experimental cases were considered, and 20,000 random iterations were performed for each experimental case.

Table 1. Range of the simulation study.

True Weibull Parameters		Number of Specimens	ECF	LCI (% of η_{true})
η_{true} (Dim.less Time)	β_{true}			
100	2	5	0.6	5
-	3	10	0.8	10
-	4	15	1.0	15
-	-	20	-	20
-	-	25	-	25
-	-	30	-	30
-	-	35	-	35
-	-	40	-	40
-	-	45	-	45
-	-	50	-	50

3. RESULTS AND DISCUSSION

From the random simulation results, the 5th, 50th and 95th percentiles ($\hat{\beta}_{5\%}$, $\hat{\beta}_{50\%}$, $\hat{\beta}_{95\%}$; $\hat{\eta}_{5\%}$, $\hat{\eta}_{50\%}$, $\hat{\eta}_{95\%}$) of 20,000 replicates of Weibull estimates could be derived for each case. The selected median estimates (i.e., $\hat{\beta}_{50\%}, \hat{\eta}_{50\%}$) were converted to the relative error ($\mathrm{RE}_{50\%}$) to represent the bias of estimators, which is defined as follows:

$$\mathrm{RE}_{50\%}(\hat{\beta}) = \mathrm{RE}(\hat{\beta}_{50\%}) = \frac{\hat{\beta}_{50\%}-\beta_{true}}{\beta_{true}}, \quad (3)$$

$$\mathrm{RE}_{50\%}(\hat{\eta}) = \mathrm{RE}(\hat{\eta}_{50\%}) = \frac{\hat{\eta}_{50\%}-\eta_{true}}{\eta_{true}}$$

In order to quantify the dispersion of estimators, a relative length of a 90% confidence interval ($\mathrm{RLCI}_{90\%}$) was utilized, which is defined as follows:

$$\mathrm{RLCI}_{90\%}(\hat{\beta}) = \mathrm{RE}(\hat{\beta}_{95\%}) - \mathrm{RE}(\hat{\beta}_{5\%}), \quad \mathrm{RLCI}_{90\%}(\hat{\eta}) = \mathrm{RE}(\hat{\eta}_{95\%}) - \mathrm{RE}(\hat{\eta}_{5\%}). \quad (4)$$

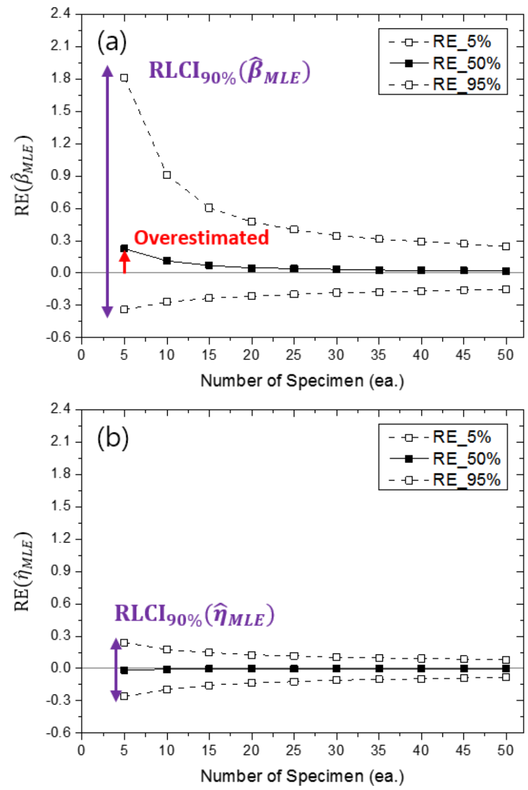

Figure 1. Effects of the number of specimens on (a) $\mathrm{RE}(\hat{\beta})$ and (b) $\mathrm{RE}(\hat{\eta})$ (β_{true}: 3; LCI: 20 % of η_{true}; ECF: 1.0).

As an example, Fig. 1 shows the effect of the number of specimens on estimation uncertainties when the other experimental factors are fixed at the certain values. It is well represented that as the number of specimens is large, estimators becomes reliable (i.e., little bias and short length of confidence interval). For estimating the shape parameter β, it is likely that the shape parameters are overestimated when the number of specimens is relatively low (i.e., $\mathrm{RE}_{50\%}(\hat{\beta}_{MLE}) > 0$). For the scale parameter η estimation, the bias is barely noticeable even when the number of specimens is low (i.e., $\mathrm{SE}_{50\%}(\hat{\eta}_{\mathrm{MLE}}) \approx 0$). It is shown that the relative length of 90% confidence interval for scale parameters (i.e., $\mathrm{RLCI}_{90\%}(\hat{\eta}_{MLE})$) are much lower than that of the shape parameter β (i.e., $\mathrm{RLCI}_{90\%}(\hat{\beta}_{MLE})$).

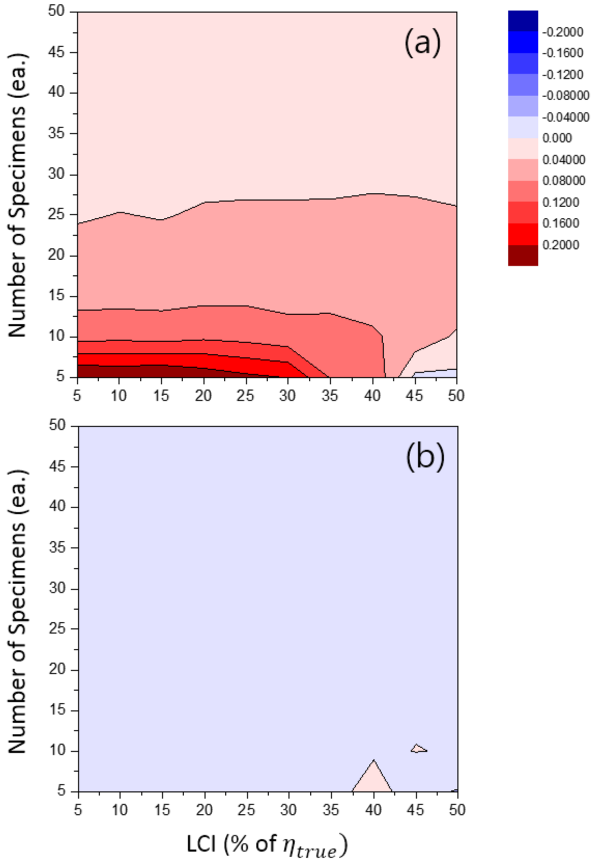

Figure 2. Effects of the number of specimens and LCI on (a) $RE_{50\%}(\hat{\beta}_{MLE})$ and (b) $RE_{50\%}(\hat{\eta}_{MLE})$ when ECF = 1.0, $\beta_{true} = 3$, and ECF = 1.0 (Park, Park et al. 2017).

Figure 2 shows the contour plots of $RE_{50\%}(\hat{\beta}_{MLE})$ and $RE_{50\%}(\hat{\eta}_{MLE})$, indicating a relative bias in the Weibull estimators. It was likely that $\hat{\beta}_{MLE}$ showed a tendency to be overestimated irrespective of the value of LCI. The unusual result that occurred in the long LCI region may not be reliable due to the low convergence ratio (Park, Park et al. 2017). Meanwhile, the value of $RE_{50\%}(\hat{\eta}_{MLE})$ was almost zero in every experimental condition.

Figure 4 shows the contour plots of $RLCI_{90\%}(\hat{\beta}_{MLE})$. As expected, the dispersion in $\hat{\beta}$ was large when the number of specimens was relatively small. In contrast, the effect of LCI was not noticeable. Additionally, some critical regions were observed, in which very wide $RLCI_{90\%}(\hat{\beta})$ were produced (Park and Bahn 2016). The gradients of $RLCI_{90\%}(\hat{\beta})$ were very high around the critical region. It is likely that this critical region was an inherent behavior of estimation uncertainty because another estimation method, which has a unity convergence ratio for the same experimental condition, also showed the critical region (Park, Park et al. 2017). Experimenters should plan the crack initiation testing so that they can avoid this critical region. When compared to the case of $RLCI_{90\%}(\hat{\beta}_{MLE})$, the overall value of $RLCI_{90\%}(\hat{\eta}_{MLE})$ was quite small. This indicates that the estimated scale parameter is more reliable than the shape parameter under the same experimental conditions

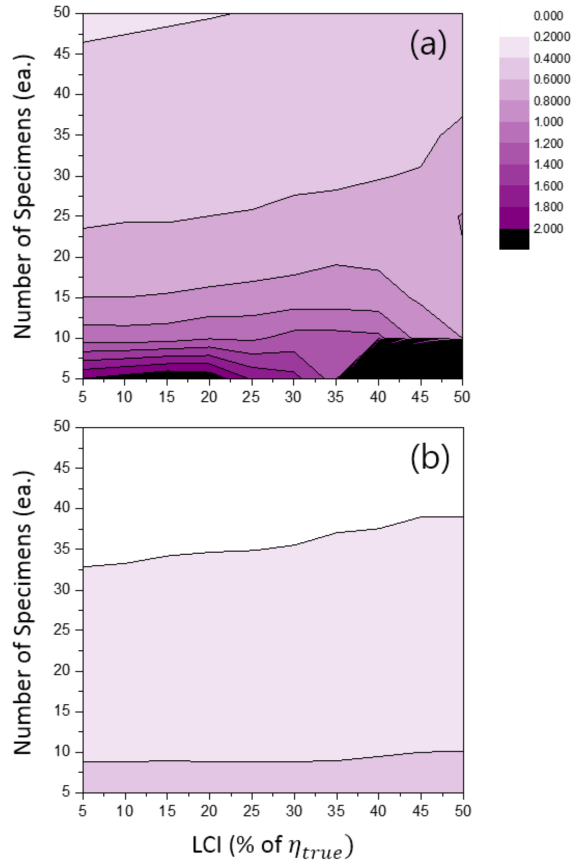

Figure 3. Effects of the number of specimens and LCI on (a) $RLCI_{90\%}(\hat{\beta}_{MLE})$ and (b) $RLCI_{90\%}(\hat{\eta}_{MLE})$ when ECF = 1.0, $\beta_{true} = 3$, and ECF = 1.0 (Park, Park et al. 2017).

4. CONCLUSION

The main goal of this study is to provide quantitative estimation uncertainties for experimenters developing a Weibull distribution model via cracking tests. The MLE method was performed with respect to the Weibull estimation. Monte Carlo simulations were used in order to quantify uncertainties estimators in various experimental conditions by considering the effects of: (1) true Weibull parameters; (2) the number of specimens; (3) end cracking fractions; and (4) length of censoring interval. The following conclusions were drawn from the study:

- In most cases, $\hat{\beta}_{MLE}$ showed a tendency to be overestimated and dispersed when the number of specimens was small and the value of ECF was low. The value of LCI does not much affect bias of $\hat{\beta}_{MLE}$. It was shown that there were critical regions, in which the dispersions were extremely large. Thus, experimenters should avoid this critical region.

- $\hat{\eta}_{MLE}$ showed almost zero bias in all simulation ranges. In most cases, the LCI did not affect the estimation uncertainty of $\hat{\eta}$. The overall bias and dispersion of $\hat{\eta}$ were much lower than those of $\hat{\beta}$ in the simulation study range. Therefore, the estimated scale parameter would be more reliable than the estimated shape parameter from the cracking tests. It was also shown that increasing the number of specimen would be more efficient to reduce the uncertainty of estimators than the more frequent censoring.

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