What are the effects of the reliability model uncertainties in the maintenance decisions?

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ABSTRACT

Most of the works proposed for the design of reliability test plans are devoted to the guaranty of the reliability performance of a product but scarce of them tackles maintenance issues. On the other hand, classical maintenance optimization criteria rarely take into account the variability of the failure parameters due to lack of data, especially when the data collection in the operating phase is expensive. The objective of this paper is to highlight through a numerical experiment the impact of the test plan design defined here by the number of the products to be tested and the test duration on the performance of a classical condition-based maintenance (CBM) policy.

1. INTRODUCTION

While the risk due to the quality and quantity of the available data is one of the major concerns in the product design and qualification processes, this issue seems not to be tackled in the operational phases and especially for the optimization of maintenance policies. Indeed, the vast majority of the works proposed in the literature concentrates on the definition of a maintenance rule for the maximization of a long-term economic profitability by assuming well-defined and stationary reliability or degradation models. However, the convergence to these stationary states can be really slow and strongly related to the knowledge level of the failure modes and therefore to the data collected during its operation. In order to reduce this uncertainty, it is necessary to integrate some of knowledge acquired during the various product qualification and endurance tests. This remains, from our point of view, one of the major areas of improvement in industrial practices, especially for the development of the condition-based maintenance approaches.

This paper should be seen as an introductive contribution to the bi-objective optimization problem of test plan and maintenance policy. The objective of the paper is to highlight through a numerical experiment the impact of the test plan defined by the number of the products to be tested and the test duration on the performance of a classical condition-based maintenance (CBM) policy.

The remainder of the paper is as follows. First, the problem statement is presented through a brief discussion on the literature review in Section 2. Section 3 will be devoted to the numerical experiment. A discussion on the challenges of combining the two approaches will be conducted in Section 4.

2. PROBLEM STATEMENT

An analysis of the current practices in industry leads us to conclude a gap between the objectives and the results in the product design phase and some of the issues within the operating context such as maintenance optimization whereas it could be recommended in general methodologies such as the Reliability-Centered Maintenance method.

The objective of the product qualification testing is to prove the ability of a given product to respect a reliability requirement at a minimal cost. This requirement is usually given through a functional specification such as a reliability level at a given time and an associated confidence level (or the value at risk). Degradation testing (Elsayed 2014) provides an efficient way for reducing both the number of products to be tested and the test duration time due to the high extrapolation capacity of the degradation models, if this is an a priori knowledge of the degradation characteristics and patterns of the degradation. This motivates the implementation of some tests on some regions rather far from the failure zone.

The construction of a maintenance policy is generally based on a set of knowledge available prior to the stat-up of the product. In case of a significant amount of experience, conventionally, stationary assumptions especially on the failure or on the degradation model could be taken for determining the policy parameters. The decision parameters can be intervals for intervention or degradation levels triggering an operation. The optimization then amounts to determining the optimal values of these decision parameters on the basis of an expected average cost criterion. Many works are developed in this framework, highlighting the interest of the CBM and PHM approaches. Generally speaking, we can say that these approaches are based on feedback that is consistent with observations or knowledge about failure or degradation models. The question is then what is the validity of such approaches in the event that this knowledge about the models and the frequency of observation in exploitation phase are limited.

Some of the works give some insights in the combination of the objectives of the two domains –optimization of the test plan and maintenance- but none of them provide a complete process for the joint optimization. Santini et al. (2014) proposes the construction of some confidence intervals for the Remaining Useful Life (RUL) based on real testing degradation data but does not propose any way in inserting in a decision maintenance framework. Hamada (2006) develops some accelerate degradation test plans to fulfill some of predetermined maintenance requirements for the periodic replacement policy. No maintenance optimization models are therefore proposed.

3. NUMERICAL EXPERIMENT

The numerical experiment that we conduct is the following. The objective is to highlight the influence of a test plan – defining here as the number of the products to test and the test duration- on the maintenance cost optimized for a product that we assume, first, that the degradation parameters are known. Then, the analysis would concern the variability of the maintenance decision criteria as a function of the uncertainty on the degradation parameters estimated through a test plan.

3.1. Degradation assumptions

In a Condition-Based Maintenance (CBM) or PHM context, a measurable degradation variable X_t is generally assumed to exist. Let assume that the evolution in time of this degradation can be modeled by an homogeneous gamma process $\{X_t, t > 0\}$ with a shape parameter a and a scale parameter b. A failure threshold x_F is supposed to be known. The product life time is then the random variable $T_f = \inf\{t > 0 | X_t > L\}$ and its distribution is given by $F(t) = 1 - \frac{IG(at, b \cdot x_F)}{\Gamma(at)}$ where $IG(x,y) = \int_y^{\infty} u^{x-1} e^{-u} du$ is the incomplete gamma function and $\Gamma(x) = \int_0^{\infty} u^{x-1} e^{-u} du$ the gamma function.

The Remaining Useful Life (RUL) of a product is defined as the duration from the current time to the failure. Under the Markovian property and the stationarity of an homogeneous gamma process, we can then extend the definition of the RUL to a function dependent on the current degradation $X_0 = x$ where the condionnal distribution is directly given by $F(t|X_0 = x) = 1 - \frac{IG(at, b \cdot (x_F - x))}{\Gamma(at)}, \forall t > 0, \forall x \geq 0.$

3.2. Test plan: Definition and results

Under the degradation assumptions proposed in the previous paragraph, the optimization of the test plan is to find the optimal tested product number n_{test} and the optimal test duration d_{test}. Let assume here that the degradation measurements are done periodically τ_{test}. Figure 1 illustrates a test for $n_{test} = 3$ products over d_{test}=1.2 time unit and a degradation measurement interval $\tau_{test} = 0.2$.

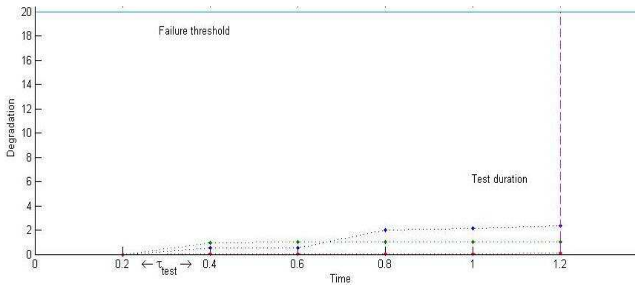

Figure 1 Example of a degradation test for 3 products

Let assume given $\theta_{test} = (n_{test}, d_{test})$. At the end of the test, we get a sample composed of the degradation increments of each product between two consecutive measurements, denoted $\{\Delta X_j^i\}$ with $i \in [\![1, \dots, n_{test}]\!]$ et $j \in [\![1, \dots, [d_{test}/\tau_{test}]]\!]$. By using the Maximum Likelihood estimation method, the degradation parameters $(\hat{a}_{\theta_{test}}, \hat{b}_{\theta_{test}})$ and, if the asymptotic normality conditions are verified, the associated multivariate distribution law can be obtained. The estimated marginal probability density functions of the degradation parameter estimators are given in Figure 2 for fixed test conditions: $n_{test} = 10$ products and 20 degradation measures every $\tau_{test} = 0.2$.

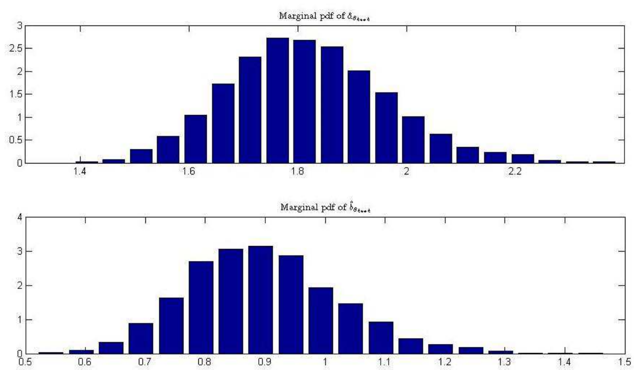

Figure 2 Estimated marginal probability density functions of the degradation estimates for $\theta_{test} = (5,10)$

In this numerical example, the values of the degradation parameter estimates are respectively $\hat{a} = 1.82$ and $\hat{b} = 0.88$.

3.3. Maintenance cost: Construction and analysis

For the shake of simplicity, we propose to analyze the performance of a preventive maintenance policy where inspections are performed every τ time. Inspections are assumed to be nondestructive and perfect. In such a case, based on the degradation assumptions, the optimal maintenance policy refers to degradation control limit policy. Let x_p be the preventive maintenance threshold and (c_i, c_p, c_c) be the respective unitary costs for inspection, preventive and corrective replacements. The optimization of the Condition-Based Maintenance (CBM) consists here in evaluation x_p^* which minimizes the expected long-run maintenance cost per unit of time which can be written within some renewal conditions as follow:

$$C_\infty\left(x_p; \tau, \hat{a}, \hat{b}\right) = \frac{c_i + c_p \int_{x_p}^{L} \pi(x)dx + c_c \int_{L}^{\infty} \pi(x)dx}{\int_0^\tau \int_0^{x_p} \pi(x) F(t|X_0 = x)dx\, dt}$$

Where:

- $(\hat{a}, \hat{b})$ are the values of the estimated degradation parameters
- $\pi(\cdot)$ the stationary deterioration density function of the maintained deteriorating product
- $F(\cdot\,|X_0 = x)$ the state-dependent RUL function given the current degradation level x.

Figure 3 sketches the evolution of the cost $C_\infty\left(x_p; \tau, \hat{a}, \hat{b}\right)$ when the preventive maintenance threshold varies from 4 to 20 given the failure threshold $x_F = 20, \tau = 2, c_i = 5, c_p = 40$ and $c_c = 100$ (all the parameters are in the unit of interest). The minimal cost is then $C_\infty^* = 9.07$ and the optimal preventive maintenance threshold $x_{opt}^* = 11$.

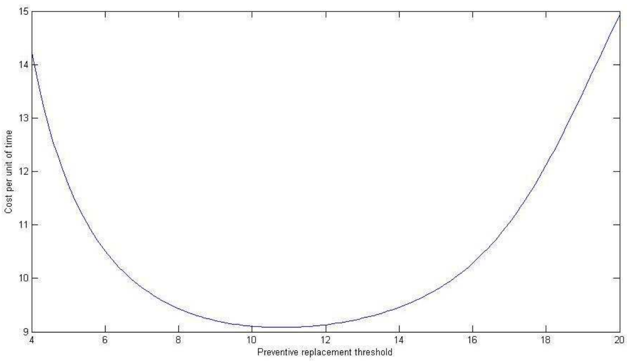

Figure 3 Maintenance cost as a function of the preventive threshold for given degradation parameters

Figure 4 gives the estimated distribution of the maintenance cost when the degradation parameters are considered unknown. This distribution is estimated by Monte-Carlo simulation. It is then possible to evaluate the risk value at the optimal cost as a function of the uncertainty on the degradation parameters due to a given test plan.

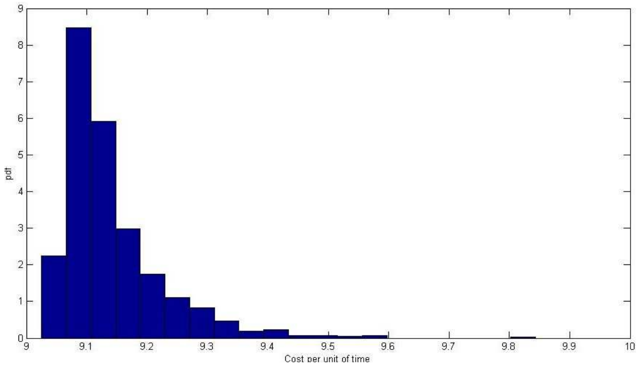

Figure 4 Estimated distribution of the maintenance cost when parameters are unknown

The numerical results provided in Table 1 highlight the influence of the number of the products to be tested and the test duration on the 90% confidence interval. Note that these figures are obtained through simulations.

Tableau 1 The 90% value at risk for different test plans

θ_{test}		Test duration d_{test}		
		5×0.2	10×0.2	15×0.2
Products	3	(9.03, 17.27)	(9.03, 10.77)	(9.04, 10.26)
	5	(9.03, 11.11)	(9.04, 10.12)	(9.05, 9.68)
	8	(9.04, 10.21)	(9.05, 9.69)	(9.05, 9.47)
	10	(9.04, 10.15)	(9.05, 9.66)	(9.05, 9.43)
	15	(9.04, 9.69)	(9.05, 9.37)	(9.06, 9.30)

No significant variations of the optimized cost C_∞^* are observed when the test plan varies, reinforcing the interest in developing some maintenance approaches not only based on the expected average cost.

4. DISCUSSION

The numerical study presented in this paper highlights some of the interests of combining the objectives of the classical design of the product reliability test plans and the associated maintenance policy optimization. We illustrate the limitation in using classical average cost criterion and the necessity to tend towards the integration of the degradation model uncertainties within the optimization process.

Robust maintenance optimization based on the Value at Risk (Chang, Pandey & van der Weide, 2016) could be a promising approach. Adding different costs in the test plan design process could be also interesting for the construction of the combined optimization process. Nevertheless, a lot of challenging issues remain such as the problem of the construction of the confidence interval for the cost because of the non-normality of the cost distribution (see Figure 4).

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BIOGRAPHIES

B. Castanier is Professor in the department of Quality, Innovation and Reliability Engineering at ISTIA/University of Angers. He received his Master degree in Statistics from the University of Montpellier (France) in 1998 and his PhD in 2001 from the University of Technology of Troyes (France). He spent 12 years as an Associate Professor in Ecole des Mines de Nantes (France). He is also the head of the SFD (Reliability Engineering and Decision-Making tools) research team of the LARIS lab of the University of Angers. His main research interests are in Reliability models and maintenance optimization.

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