Selective Maintenance under Uncertainty

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ABSTRACT

In this paper, a new multi-objective robust selective maintenance model is formulated by considering the uncertainties produced by imperfect inspections. The resulting optimization problem aims at maximizing the expectation of the probability of a repaired system completing a mission and simultaneously minimizing its variance. A multi-objective particle swarm optimization algorithm in introduced to identify the Pareto front, which offer a set of non-dominated selective maintenance strategies. An illustrative example shows that the selective maintenance strategy with the maximum expectation of the probability of a repaired system completing a mission may not be desirable as it usually possesses a huge variance. Several comparative studies are also conducted to examine the effect of observation accuracy and maintenance budget on the results. It concludes that the proposed approach can effectively improve the robustness of the probability of a repaired system completing a mission.

1. INTRODUCTION

Maintenance is a crucial way to keep systems away from failures, restore system performance, and prolong the systems' remaining life. Many systems in military and industry are required to perform a sequence of missions with breaks between two adjacent missions. Maintenance activities can only be carried out in the break between two adjacent missions. In addition to the failure behaviors of components in the system, the probability of the system successfully completing the next mission relies heavily on the efficiency of maintenance activities executed in the breaks. Unfortunately, maintenance resources to be consumed, such as, maintenance budget, time, and manpower, are oftentimes potentially limited during a break, and therefore, performing all the desirable maintenance activities becomes impossible. In such cases, only a subset of maintenance activities among all the feasible maintenance activities can be chosen to ensure the completion of the succeeding mission, and it is the so-called selective maintenance (Rice, Cassady, and Nachlas, 1999; Cassady, Pohl, and Jr, 2001).

Existing reported work on selective maintenance strategies assumed that the health status of all the components, i.e., states (e.g., functioning or failed) and effective ages, can be exactly known in advance. However, due to the inaccurate sensing technologies or vague judgements from engineers, observations on components' health status may be imperfect. Such imperfection of observations introduces additional uncertainty to reveal the true values of components' states and effective ages, and will eventually propagate to the probability assessment of a repaired system successfully completing the next mission. On the other hand, it may be unreliable to choose a selective maintenance strategy which maximizes the expectation value of the probability of a system successfully completing the next mission, as the strategy with less uncertainty associated with the concerned probability value is preferred. In this work, a new robust selective maintenance strategy is proposed to find optimal maintenance actions for all the components under uncertainty. The proposed method is demonstrated via a numerical example.

2. POSTERIOR DISTRIBUTIONS OF STATE AND EFFECTIVE AGE

To quantify the uncertainty of imperfect observations, the observation probability matrix and the condition probability density function (PDF) are introduced in this paper to represent the stochastic relations between the observed values and the true values of components' states and effective ages, respectively. As a consequence, the Bayes' rule is used to determine the posterior probability distribution of a component being in the certain functioning/failure state with each imperfect observed state. If the component remains functioning at the end of a mission, the effective age is obviously a certain value. However, if the component is failed before the break, the uncertainty of effective age can be expressed by the truncated PDF of failure time and the posterior PDF of the true value when the component is observed in functioning and failure state, respectively.

3. COMPONENT'S SURVIVAL PROBABILITY WITH IMPERFECT MAINTENANCE

3.1. State and Effective Age with Imperfect Maintenance

An imperfect maintenance model, the Kijima type II model specifically, is used to quantify the efficiency of various maintenance activities. In this work, the maintenance efficiency, i.e. age reduction factor, is related with the maintenance cost allotted to a component in the break. For a functioning component, a greater value of maintenance cost represents a more efficient preventive maintenance action. On the other hand, for a failed component, a minimal repair can only bring the component back to a functioning state, but no decrease to the effective age. Consequently, the maintenance action of a failed component can be separated into two progressive stages (Pandey, Zuo, and Moghaddass, 2013): (1) a failed component is put into the functioning order first by the minimal repair; (2) an additional maintenance action is performed to further decrease the effective age of the functioning component restored by the minimal repair.

3.2. Components' Survival Probability Assessment

The uncertainties of components' state and effective ages introduced by imperfect observations will propagate to the component's survival probability in the next mission. In this study, the uncertainty of a component's survival probability is quantified by the expectation and the variance of the component's survival probability.

4. PROBABILITY OF SYSTEM SUCCESSFULLY COMPLETING THE NEXT MISSION

With the known structure function of a studied repairable system, the probability of system successfully completing the next mission can be calculated by components' survival probability at the end of the next mission. Consequently, the uncertainty associated with the probability of the system completing the next mission is quantified by the expectation and the variance of the probability. For a system connected by n components in series, the variance of the probability of system successfully completing the next mission is expressed as (Coit, 1997):

$$Var\{R_{\text{Series}}(t)\} = \prod_{i=1}^{n} \left[R_{i}^{E}(t)^{2} + Var\{R_{i}^{E}(t)\} \right] - \prod_{i=1}^{n} R_{i}^{E}(t)^{2} \quad (1)$$

where $R_i^E(t)$ and $Var\{R_i^E(t)\}$ represent the expectation and variance of the survival probability for component *i*. Likewise, for a system connected by *n* components in parallel, the variance is calculated as:

$$Var\{R_{\text{Parallel}}(t)\} = \prod_{j=1}^{n} \left[(1 - R_{i_j}^{E}(t))^2 + Var\{R_{i_j}^{E}(t)\} \right] - \prod_{j=1}^{n} (1 - R_{i_j}^{E}(t))^2$$
(2)

5. ROBUST SELECTIVE MAINTENANCE MODEL

The selective maintenance problem aims at determining the maintenance cost optimally allocated for components in the break. Consequently, a multi-objective robust selective maintenance model is formulated to maximize the expectation of the probability of a repaired system completing the next mission and simultaneously minimize its variance, expressed as:

$$\max \left\{ R_{s,k+1}^{E}(z_{k+1}), -Var\{R_{i,k+1}(z_{k+1} | u_{i,k+1})\} \right\}$$

s.t.
$$\sum_{i=1}^{M} C_{i,k} \leq C^{0}$$

$$c_{i,k} \leq c_{i}^{rp}$$

$$-c_{i,k} \leq 0$$

$$i = 1, 2, \cdots, M$$

(3)

where $R_{s,k+1}^{E}(z_{k+1})$ and $Var\{R_{i,k+1}(z_{k+1} | u_{i,k+1})\}$ represent the expectation and the variance of the probability of the system successfully completing the (k+1) th mission, respectively; z_{k+1} is the length of the (k+1) th mission duration; C^{0} is the maintenance budget; $C_{i,k}$, $c_{i,k}$, and c_{i}^{TP} denote the total maintenance cost, the corrective/preventive maintenance cost, and the replacement cost of component i, respectively.

In this work, the resulting optimization problem will be resolved by a multi-objective particle swarm optimization algorithm to identify the Pareto front, which can offer a set of non-dominated selective maintenance strategies.

6. CASE STUDY

A simple series-parallel system consisting of four components, as shown in Figure 1, is used to demonstrate the advantages of the proposed method. The system has just completed the k th mission and will continue to execute the (k + 1) th mission after a break.



Figure 1 Series-parallel system

Given the maintenance budget C_0 =\$80,000, the Pareto front can be identified as plotted in Figure 2. The 95% bounds of all Pareto solutions are furtherly depicted in

Figure 3. It can be seen that the lower bound of the solution with the maximal expectation is too small, and the upper bound of the solution with the minimal variance is small too. Therefore, both the two solutions are not preferable. Several comparative studies are also conducted to examine the effect of observation accuracy and maintenance budget on the results. It concludes that the proposed approach can effectively improve the robustness of the probability of a repaired system completing a mission.



Figure 3 The 95% bounds of Pareto solutions

7. CONCLUSION

In this paper, a new multi-objective robust selective maintenance model is formulated to take account of the imperfection of observations. The optimization problem aims at maximizing the expectation and simultaneously minimizing the variance. A multi-objective particle swarm optimization algorithm in introduced to identify the Pareto front, which can offer a set of non-dominated selective maintenance strategies. An illustrative example shows that the selective maintenance strategy with the maximum expectation of the probability of a repaired system completing a mission may not be desirable as it usually possess a huge variance. Several comparative studies are also conducted to examine effectiveness of the proposed method. It concludes that the proposed approach can effectively improve the robustness of the probability of a repaired system completing a mission.

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Yu Liu is a Professor in the School of Mechatronics Engineering, at the University of Electronic Science and Technology of China. He received his PhD degree in Mechatronics Engineering from the University of Electronic Science and Technology of China in 2010. He was a Visiting Pre-Doctoral Fellow in the Department of Mechanical Engineering at Northwestern University, Evanston, U.S.A. from 2008 to 2010, and a Postdoctoral Research Fellow in the Department of Mechanical Engineering, at the University of Alberta, Edmonton, Canada from 2012 to 2013. His research interests include system reliability modeling and analysis, maintenance decisions, prognostics and health management, and design under uncertainty.

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