# Model- and Non-model-based Damage Detection Methods Using Vibration Data

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## ABSTRACT

Modal parameters of a structure, including natural frequencies, modal damping ratios and mode shapes, are directly related to its material properties, such as mass, damping and stiffness. Changes in material properties of a structure due to occurrence of damage can result in those in its modal properties. In this work, model- and non-modelbased structural damage detection methods by use of modal parameters are introduced. The model-based method uses changes in natural frequencies of a structure; it requires an accurate physics-based model of the structure and a minimum amount of vibration measurement. The nonmodel-based methods use measured mode shapes and curvature mode shapes, and they do not require use of structural models. Experimental damage detection on a space frame, a beam and a plate are carried out to validate the model- and non-model-based methods.

## **1. INTRODUCTION**

Model-based damage detection can be divided into two problems, including the forward problem and inverse problem [1]. The forward problem refers to the calculation of changes in natural frequencies caused by damage, and the inverse problem refers to detection of locations of damage from the changes in natural frequencies. Since only natural frequencies are used in the modal-based method, their changes are calculated by comparing measured natural frequencies to predicted ones from a finite element model. Both changes in natural frequencies caused by damage and those due to modeling error in a finite element model and measurement noise are included in the forward problem. Hence, modeling error must be minimized so that changes in the natural frequencies due to the error are smaller than those caused by damage [1]. There exist many approaches to formulate and solve the inverse problem. A trust-region optimization method called Levenberg-Marquardt method is used in this work to detect loosening of bolted joints in a space frame.

The non-model-based methods can detect damage in beams and plates. While mode shapes of undamaged beams and

plates are usually not available, they can be well approximated by those from polynomial fits. Comparing mode shapes and curvature mode shapes of damaged beams and plates with those from polynomial fits yields associated damage indices, and damage can be detected near neighborhoods with high damage indices. The non-modelbased methods were applied to detecting embedded horizontal cracks in a beam and damage in the form of thickness reduction in a plate.

# 2. MODEL-BASED METHOD

This section briefly describes the inverse problem in the model-base damage detection method. LM method is then applied to detecting loosening of bolted joints in a space frame. Modeling methodologies for bolted joints and fillets in a space frame can be found in Refs. [2,3], and details regarding Levenberg-Marquardt method can be found in Ref. [4]

# 2.1. Inverse Problem

The inverse problem in model-based damage detection can be formulated as a nonlinear least-square problem. Consider a cantilever beam in Fig. 1 for example. The beam is equally divided into n sections. The stiffness of the i -th section is represented by a non-dimensional stiffness  $G_i$ with  $0 < G_i \le 1$ , and  $G_i = 0$  and  $G_i = 1$  correspond to *i* -th section being fully damaged and undamaged, respectively. The entire beam's stiffnesses can be denoted by  $G = [G_1 \ G_2 \dots \ G_n]^T$ . Assuming that mass of the beam does not change at all due to damage, m calculated natural frequencies of the beam are nonlinear functions of G and can be denoted by  $\boldsymbol{\lambda} = [\boldsymbol{\lambda}_1(G) \boldsymbol{\lambda}_2(G)..., \boldsymbol{\lambda}_m(G)]^T$ ; m measured natural frequencies are denoted by  $\lambda^d = [\lambda_1, \lambda_2, ..., \lambda_m]$ . The inverse problem is to find changes in each entry of G that minimize the difference between the calculated and measured natural frequencies. Since the number of stiffness parameters is usually greater than that of calculated/measured natural frequencies, the inverse problem is an under-determined nonlinear least-square proble, which can be expressed by

$$\min Q(G) = \frac{1}{2} \sum_{j=1}^{m} [\boldsymbol{\lambda}_{j}(G) - \boldsymbol{\lambda}_{j}^{d}]^{2} = \frac{1}{2} \sum_{j=1}^{m} r_{j}^{2} = \frac{1}{2} r^{T} r \quad (1)$$

where  $r = [r_1 \ r_2 \dots r_m]^T$  and  $r_j = \boldsymbol{\lambda}_j(G) - \boldsymbol{\lambda}_j$ . The solution of

the inverse problem G contains the information about the locations and extent of damage.



Fig. 1 Schematic of a cantilever beam whose stiffness is represented by a set of non-dimensional stiffness parameters

#### 2.2. Model-based Damage Detection

A three-bay aluminum space frame was fabricated with thin-walled L-shaped beams and bolted joints. It consisted of 12 L-shaped thin-walled beams and 14 beams with a rectangular cross-section. The beam members were connected by 12 bolted joints with multiple bolted connections in each joint. The frame was bolted on a plate through four cubic blocks. The plate was bolted to the ground through four bolts, as shown in Fig. 2(a). With the development of the modeling techniques for fillets in thinwalled beams [2] and for bolted joints [3], an accurate physics-based model for the space frame structure is developed with a reasonable number of degrees of freedom. In the inverse problem, each beam member, bolted joint and bolts on the bottom is treated as an element group and totally 39 groups are obtained as a result; the numbering of element group members is shown in Fig. 2(a).



Fig. 2 (a) Finite element model of a three-bay space frame and (b) damage detection results of the frame with bolts in group 9 loosened.

The model-based damage detection method is used to detect bolt loosening in the space frame. The structure is divided into 39 groups (Fig. 2(a)). Each solid cylinder that models a bolted connection connecting the bottom plate to the ground is represented by a group (groups 1-4). The bottom plate is represented by a group (group 5). Each of the L-shaped beams and the horizontal and diagonal beams is represented by a group (groups 6-8, 11-13, 16-18, 21-23, 26-39). The eight solid cylinders representing the eight bolted connections that connect the L-shaped beams and the bracket, in each of the eight bolted joints in the middle of the structure, are grouped together (groups 9, 10, 14, 15, 19, 20, 24, 25). Eight bolted connections in a bolted joint were loosened to hand-tight. The first 10 measured natural frequencies were used to detect the damage, and loosening could be detected in both numerical and experimental damage detection (Fig. 2(b)).

#### **3. NON-MODEL-BASED METHODS**

Curvature mode shapes of beams and plates have been extensively used for damage detection [5,6]. In practice, measurement noise can be amplified in calculated curvature mode shapes, and damage detection based on the calculated curvature mode shapes can be compromised. This section briefly describes a non-model-based method for damage detection method for beams with embedded horizontal cracks, and the method is extended to detect damage in plates.

# 3.1. Damage Detection for Beam Structures.

A curvature mode shape of a beam structure at point i, denoted by  $y''_i$ , can be calculated by

$$y_i'' = \frac{y_{i+m} + y_i + y_{i-m}}{(mh)^2}$$
(2)

where  $y_i$  is the mode shape at point *i*, *h* is the distance between two neighboring measurement point and *m* is a resolution parameter. One can alleviate adverse effects of measurement noise on calculated curvature mode shapes by increasing *m*. Figure 3 shows calculated curvature mode shapes with different *m*, and it can be seen that measurement noise is dominant in calculated curvature mode shapes when *m* is small, and the noise becomes smaller as *m* increases.



Fig. 3 Calculated curvature mode shapes of a beam with an horizontal embedded crack with (a) m=1, (b) m=5, (c) m=10 and (d) m=15.

Comparing a curvature mode shape of a damaged beam structure with that of an undamaged beam structure can yield a curvature damage index, which is defined by

$$\boldsymbol{\delta}_{i} = \left(y_{i}'' - y_{i}^{u''}\right)^{2} \tag{3}$$

where  $y_i^{u''}$  is a curvature mode shape of the undamaged beam at point *i*. However, curvature mode shapes of undamaged beams are usually unavailable in practice. It is proposed that a mode shape of an undamaged structure be approximated by one from a polynomial that fits a mode shape of a damaged beam with a properly determined order. More details on this approximation can be found in Ref. [5].

A uniform acrylonitrile butadiene styrene cantilever beam with an embedded horizontal crack was fabricated with dimensions shown in Fig. 4(a), and its third mode shape was measured using non-contact operational modal analysis. Non-model-based damage detection result is shown in Fig. 4(b), and it can be seen that ends of the crack are indicated in the neighborhoods of high curvature damage indices defined in Eq. (3).



Fig. 4 (a) Dimensions of a uniform acrylonitrile butadiene styrene cantilever beam with an embedded horizontal crack and (b) curvature damage index associated the third mode shape of the beam in (a).

### 3.2. Damage Detection for Plate Structures.

Damage in a plate structure can be detected by comparing a mode shape of a damaged structure with that of an undamaged structure by use of a mode shape damage index, which can be defined by

$$\boldsymbol{\delta}(x,y) = \left[z(x,y) - z^{u}(x,y)\right]^{2} \tag{4}$$

where z and  $z^{u}$  are mode shapes of the damaged and undamaged structures at point (x, y). However, mode shapes of an undamaged structure are not available in practice. It is proposed that a mode shape of an undamaged plate be approximated by one from a polynomial that fits a mode shape of a damaged plate structure with a properly determined order, which is an extension of that for beams. More details on this approximation can be found in Ref. [6]. An experiment was conducted to detect damage in the form of thickness reduction in an aluminum plate structure. Dimensions of the plate and its damage are shown in Fig. 5(a). Thickness of the undamaged portion of the plate is 4.75 mm, and that of the thickness reduction is 0.5 mm. The plate was excited using a speaker, as shown in Fig. 5(b). The mode shape of the damage plate at the frequency of 1350 Hz was measured, as shown in Fig. 6(a), and the mode shape damage index of the plate, as defined in Eq. (4), is shown in Fig. 6(b). The damage can be detected in neighborhoods of high mode shape damage indices.



Fig. 5 (a) Dimensions of an aluminum plate with damage in the form the thickness reduction and (b) experimental setup for mode shape measurement of the plate using noncontact acoustic excitation.



Fig. 6 (a) Dimensions of an aluminum plate with damage in the form the thickness reduction and (b) experimental setup for mode shape measurement of the plate using noncontact acoustic excitation.

### 4. CONCLUSION

Modal parameters can be used to detect damage in a structure, as they are directly related to its material properties that can be changed by damage. Model- and nonmodel-based damage detection methods have been discussed. The former requires an accurate physics-based model, while the latter do not. The inverse problem in the model-based damage detection method has been described. Experimental damage detection using natural frequencies and mode shapes were conducted, and damage in a space frame, a beam and a plate were successfully detected.

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# BIOGRAPHIES

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