A Fresh New Look on System-level Prognostic: Handling Multi-component Interactions, Mission Profile Impacts, and Uncertainty Quantification

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Abstract

Model-based prognostic approaches use first-principle or regression models to estimate and predict the system's health state in order to determine the remaining useful life (RUL). Then, in order to handle the prediction results uncertainty, the Bayesian framework is usually used, in which the prior estimates are updated by infield measurements without changing the model parameters. Nevertheless, in the case of system-level prognostic, the mere updating of the prior estimates, based on a predetermined model, is no longer sufficient. This is due to the mutual interactions between components that increase the system modeling uncertainties and may lead to an inaccurate prediction of the system RUL (SRUL). Therefore, this paper proposes a new methodology for online joint uncertainty quantification and model estimation based on particle filtering (PF) and gradient descent (GD). In detail, the inoperability input-output model (IIM) is used to characterize system degradations considering interactions between components and effects of the mission profile; and then the inoperability of system components is estimated in a probabilistic manner using PF. In the case of consecutive discrepancy between the prior and posterior estimates of the system health state, GD is used to correct and to adapt the IIM parameters. To illustrate the effectiveness of the proposed methodology and its suitability for an online implementation, the Tennessee Eastman Process is investigated as a case study.

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1. Introduction

Research in the field of failure prognostic, in the literature, is conducted generally at component-level (Daigle, Bregon, & Roychoudhury, 2012; Atamuradov, Medjaher, Dersin, Lamoureux, & Zerhouni, 2017). However, complex engineering systems are composed of multiple individual components operating interactively. Thus, when one or more components fail, the performances of the whole system are adversely affected. Therefore, the development of system-level prognostic approaches is also essential. However, in that perspective, several challenges are faced. Among them, three main challenges will be investigated and solved in this paper.

The first challenge concerns the development of the model that allows the various factors influencing the evolution of system degradation to be taken into account, including the components' mutual interactions and the effects of the mission profile. However, most systems are composed of heterogeneous elements with different operating mechanisms, then modeling them becomes difficult tasks (Liu & Zio, 2016).

The second challenge is related to uncertainty quantification. Indeed, the transition from component-level to system-level prognostic leads to an increase in the number of uncertainty sources, which causes more issues when predicting the SRUL (Das, Elburn, Pecht, & Sood, 2019).

The third challenge concerns the online implementation of the prognostic algorithms. This is due to two principal reasons: 1) the unavailability of prior and extended knowledge about the systems under study because of the impossibility to perform run-to-failure experiments for equipment availability, cost, or safety reasons (Acuña & Orchard, 2017), and 2) the implementation of these algorithms requires high computing resources, given the modern system complexity.

In this paper, a methodology for online failure prognostic at the system-level is presented. The inoperability input-output model (IIM), which considers component interdependencies, mission profile, and inner component degradations, is used as a modeling framework. This methodology requires minimal input information on system degradation since the parameters of the IIM model can be estimated and corrected online using our developed algorithm based on gradient descent. A particle filter then exploits the resulting IIM model to estimate the system's health state by considering the process uncertainty and the monitoring data received from the sensors. Once a fault is detected, and based on the system's functional architecture, its estimated health state is propagated into the future to determine its system remaining useful life (SRUL). This methodology's results are evaluated, at each execution of the dedicated algorithm, to find a balance between prediction accuracy and computation time.

The remainder of the paper is organized as follows. Firstly, Section 2 stated the problem of failure prognostic at system-level and the considered assumptions in this paper. Section 3 presents the inoperability input-output model. Section 4 focuses on the description of the elements composing the proposed methodology, and its online implementation is detailed in Section 5. The proposed methodology's effectiveness and applicability are discussed in Section 6, through a real industrial case study, which is the Tennessee Eastman Process. Finally, Section 7 concludes the paper and gives some future works.

2. PROBLEM STATEMENT AND ASSUMPTIONS

This section presents the assumptions taken in this study, and the requirements on data and expertise are highlighted. To do that, let us first describe the context of the use of the proposed methodology for the SRUL prediction.

In this paper, we are interested in systems composed of M components that interact between them (differently depending on the system architecture) and the environment. Each component is assumed to have a failure mode caused by its own degradation and the degradation impacts of other components that interact with it. A component fails when it reaches a supposed known threshold. The degradation of each component i is monitored by a relevant sensor that provides noisy measurements y_i. The sensors are selected based on the failure mechanisms to track in time.

The degradation of a system is characterized by its inoperability, which is expressed at k-*th* instant by a vector q_k containing each component's inoperability. The inoperability q_i^k of a component i represents the decrease in its health state compared to the nominal one at the initial instant (k_0).

In light of the context of use described above, the input data needed to implement a prognostic approach at the system-level is :

- Failure threshold of the system components;
- The system architecture;
- The online health indicator value of the system's components provided by sensors and their distributions;
- The degradation trends of the system's components with their uncertainty

However, for efficiently deploying a system-level prognostic approach in practice, including the one detailed in this paper, it is necessary to perform the system's functional and dysfunctional analysis. Particularly, for the case of hidden degradation, the proposed methodology could be performed after accomplishing the tasks presented below for construction of health indicator.

2.1. Functional and Dysfunctional System Analysis

For modeling a complex system, it is necessary to study its behavior, perform functional and dysfunctional analyses and deploy an effective monitoring process to acquire useful data, Figure 1. These tasks provide the following essential information:

1. **Identification of critical components to be monitored.** In general, systems have numerous elements interacting with each other. Still, not all of them are critical, i.e., significantly contribute to the evolution of the system degradation process, and thus do not warrant increased monitoring of their health state. Thus, a selection of critical components to be monitored must be made before implementing any monitoring process. This can be achieved by using risk analysis and dependability methods (Brahimi, Medjaher, Leouatni, & Zerhouni, 2017; Sarih, Tchangani, Medjaher, & Pere, 2018).
2. **Selection of physical parameters to monitor.** After locating the critical components, the system expert should identify the appropriate physical parameters (i.e. health indicators) to monitor. These parameters are selected based on the experience feedback gathered during the system exploitation. Therefore, it is necessary to know the failure modes that may affect the system components. Thus, depending on these modes, choose one or more parameters to monitor (Mosallam, Medjaher, & Zerhouni, 2015).
3. **Sensor selection**. After selecting the parameters representing the degradation process, it is necessary to choose the appropriate sensors to record representative data. The criteria for selecting sensors for monitoring the system health state should consider six aspects: parameters to be monitored, reliability, accuracy, span, resolution, characterizes properties, and cost (Cheng, Azarian, & Pecht, 2008).

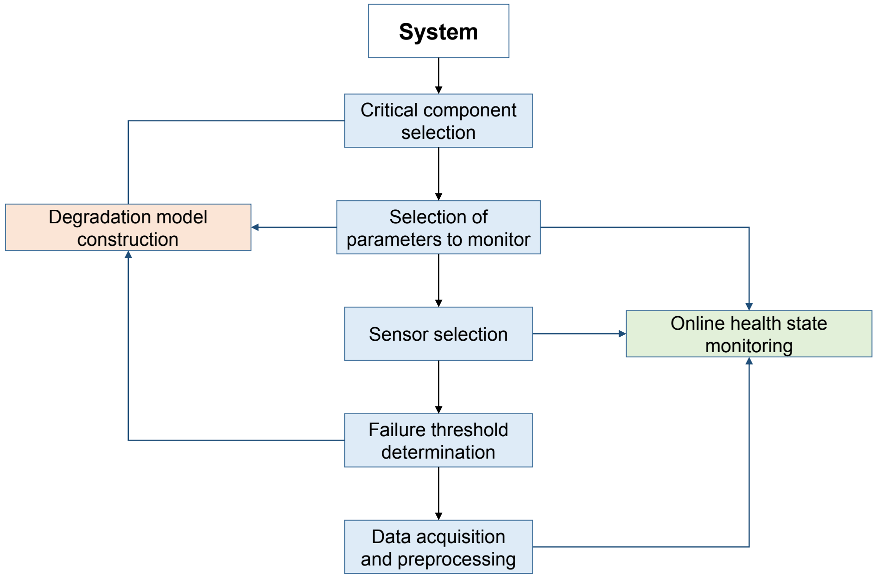


Figure 1. Prior tasks for system degradation modeling.

4. **Failure threshold determination.** The problem of determining failure thresholds remains a key hurdle for the deployment of any prognostic approach. In practice, several ways, such as statistical or expert knowledge-based, can be utilized to set the failure threshold. First, statistical methods are either experimental (run-to-failure experiments) or simulation-based (if the degradation phenomenology is known and modeled). The obtained failure threshold can be a single value (if only one simulation/experiment is conducted) or distribution if several experiments/simulations are performed (which corresponds to the notion of hazard zone (Saxena, Celaya, Saha, Saha, & Goebel, 2010; Tang, Orchard, Goebel, & Vachtsevanos, 2011)). Second, failure thresholds can be chosen by the system's designers, experts, or operators for safety or operational reasons. Finally, they can be found in standards.

5. **Data acquisition and pre-processing.** Once the sensors are chosen and installed on the components to be monitored, the corresponding signals are first pre-processed before using them for prognostic purposes. Data preprocessing involves data cleaning, for errors/noise cancellation and data analysis, for in-depth interpretation (Gouriveau, Medjaher, & Zerhouni, 2016).

In the case of visible degradation where the degradation state can be directly measured, the acquired representative and reliable data can be used to build degradation models for failure prognostic. Contrarily, for hidden degradation mechanisms, it is necessary to apply the health indicator construction methods (Nguyen & Medjaher, 2021) before deploying the proposed methodology in this paper. The following section describes in detail the system degradation modeling using a tool named: the inoperability input-output model.

3. SYSTEM DEGRADATION MODELING FRAMEWORK

Confronted with the nonexistence of a modeling framework to represent the system degradation in a comprehensive way, the authors have proposed in previous works the inoperability input-output model (IIM) (Tamssaouet, Nguyen, & Medjaher, 2019). This model considers heterogeneous systems by introducing the concept of inoperability, which expresses the distance between the state of health of the current system and its failure threshold. The fact that the IIM can take into account mutual interactions between numerous elements offers a promising perspective when applying it in the PHM domain. The formulation of the IIM, in the context of prognostic, is as follows (Tamssaouet et al., 2019):

$$q(t) = \kappa(t).[A.q(t-1) + c(t)], \tag{1}$$

where:

- $q(t)$ is a vector representing the overall inoperabilities of the system components at time t;
- A is a matrix representing the multi-dimensional interdependencies between the system components;
- $c(t)$ represents the internal inoperabilities of the system components at time t;
- $A.q(t)$ represents the inoperabilities of the components due to their interdependencies;
- $\kappa(t)$ is a diagonal matrix representing the environment or mission profile effects on the component inoperabilities at time t.

As it can be seen in Eq. (1), the degradation of a component i, characterized by an inoperability $q_i(t)$, depends on its inherent natural degradation mechanisms expressed by $c_i(t)$ and on the degradation induced by the interactions with other components through the matrix A. Concerning the influence factor $\kappa(t)$, it represents the dynamics of the degradation evolution, accelerating or reducing it, with respect to the environmental and the operating conditions. The proposed IIM can address a wide range of interdependencies between the system components and several situations related to systems operation (Tamssaouet, Nguyen, Medjaher, & Orchard, 2020).

Discussion of the IIM parameters

3.1. Inoperability

It corresponds to a column vector of inoperabilities of the n components of the system at time t:

$$q(t) = [q_i(t)]_{n\times 1} \quad ; \quad \forall i = 1, 2, \ldots, n \tag{2}$$

Definition: The inoperability of a component $q_i(t)$ represents the decrease of its performance compared to its flawless state (non-degraded performance). In practice, the component performance can be related to its precision, its stability, etc. It is expressed as:

$$q_i(t) = \frac{|performance_i(t_0) - performance_i(t)|}{performance_i(t_0)} \tag{3}$$

For prognostic, and as shown in Figure 2, the inoperability can be interpreted as the ratio between G (distance between the system current state from its initial state) and H (distance between the initial state and the failure threshold). Furthermore, the inoperability holds the properties presented below.

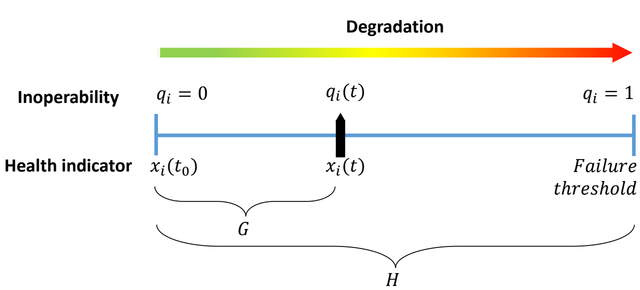


Figure 2. Schematic representation of the inoperability concept.

Inoperability properties:

- The inoperability of each component is a unique value between 0 and 1.
  - $q_i(t) = 0$: the component i is healthy (with an ideal performance);
  - $q_i(t) = 1$: the component i is considered faulty, i.e., the component has reached its failure threshold.
- In general, at the initial state, we have $t_0 = 0$ and $q_i(t_0) = 0$.

The inoperability of each component can be obtained by monitoring a health indicator (extracted from sensor signals) or a function combining several health indicators (using data fusion techniques). For the calculation of a component inoperability from its monitored health indicator, refer to (Tamssaouet et al., 2019).

3.2. Matrix of Interdependencies

This matrix formalizes the different interdependencies between the system components.

$$A = [a_{ij}]_{n\times n} \quad ; \quad \forall\, i, j = 1, 2, \ldots, n \tag{4}$$

Each component a_{ij} of the matrix corresponds to the influence of the inoperability of a component j on the inoperability of a component i.

Table 1. Degradation influence between multiple components.

case	Description
$a_{ij} = 0$ and $a_{ji} = 0$	Component j and i are independently subject to gradual degradation.
$a_{ij} > 0$ and $a_{ji} = 0$	Component j influences unilaterally the degradation behavior of component i.
$a_{ij} > 0$ and $a_{ji} > 0$	Components j and i influence each other.

Properties of matrix A:

- A is a square matrix $n \times n$ where n is the number of components;
- The IIM can handle negative values of a_{ij} for the cases where the degradation of one component slows down the degradation of other system components. However, we focus on the more common and realistic cases where $a_{ij} \geq 0$ (McCall, 1965), i.e., when a component j is degraded, it does not affect ($a_{ij} = 0$) or accelerate ($a_{ij} > 0$) the degradation of a component i;
- $a_{ij} = a_{ji} = 0$ means that there is no interaction between the components i and j; and $a_{ij} = 0.5$ means that the inoperability of a component i is increased by half of the inoperability of a component j;
- When $i = j$, $a_{ij} = 0$ because it is considered that the inoperability of a component does not affect the component itself;

- The bigger a_{ij} is, the greater is the influence of j on i.

3.3. Matrix of Influence Factors

As all systems interact with their environment, it is necessary to take into account the environmental conditions when considering the evolution of the system's health state. These conditions consist of environmental parameters (ambient temperature, humidity, etc.) or operating conditions, also called mission profile (setpoints, load durations, production loads, etc.), and affect the system during the major phases of its life cycle. In our model, these influence factors are represented by the matrix κ:

$$\kappa(t) = diag[\kappa_i(t)]_{n\times n} \tag{5}$$

where κ_i is specific to each component. Without loss of generality, κ_i is assumed to be positive.

The added value provided by this factor is its variation over time, depending on the changes in the operating or environmental conditions. The meaning of the different values of κ_i is explained in Table 2.

Table 2. Signification of the influence factor κ.

<table>
<tr><th></th><th>Inoperability</th><th>Meaning</th></tr>
<tr><td>$\kappa_i = 0$</td><td>q_i is stationary</td><td>The component does not degrade.</td></tr>
<tr><td rowspan="3">$\kappa_i > 0$</td><td rowspan="3">q_i varies over time</td><td>$\kappa_i = 1$: Normal case when a system operates in a normal condition with a normal work load.</td></tr>
<tr><td>$0 < \kappa_i < 1$: When a system operates in a favorable environment or with a low work load, its degradation processes are slower than in the normal case.</td></tr>
<tr><td>$\kappa_i > 1$: Accelerated degradation due to a hostile environment or a high work load.</td></tr>
</table>

In Table 2, when $\kappa_i = 1$, it is considered that the environment has no influence on the component i at time t. Indeed, this means exactly that the inoperability of a component i is only due to its internal degradation and the degradation induced by other components.

It should be noted that here the interpretation of factor κ differs from the one initially proposed in (Haimes & Jiang, 2001), where it expresses the restoration of the operability of a system. In our work, the factor κ is used to take into account the effects of a mission profile on the evolution of system degradations. As shown in Figure 3, the variation of κ_i will accelerate or decelerate the original degradation of a component i.

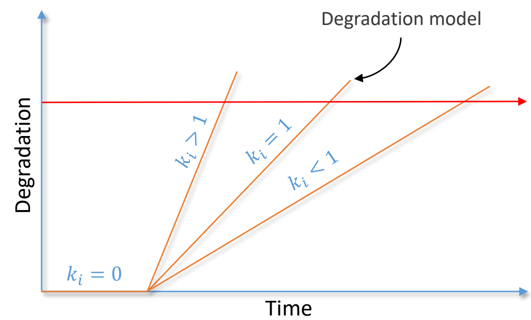


Figure 3. Degradation model variation in function of influence factor values.

4. METHODOLOGY FOR JOINT PARAMETER ESTIMATION AND SRUL PREDICTION

The methodology proposed in this paper for the online determination of the SRUL involves three steps as shown in Figure 4. The first one consists of the determination of the system degradation model parameters (i.e., IIM). Once this model is determined, the second step concerns its utilization to estimate the system health state and predict its future evolution while characterizing the related uncertainties. This step is carried out online by combining model predictions and monitoring data. The third step is the calculation of the SRUL based on the system configuration. These steps will be detailed in this section, while its online application will be presented in Section 5.

4.1. Estimation of System Degradation Model Parameters

In a model-based prognostic approach, data are mainly used to identify and update a pre-determined degradation model's parameters. In the literature, there exist numerous methods that can be applied for parameter estimation. Among them, the gradient descent (GD) method (Snyman & Wilke, 2018) is proposed for this work. Indeed, this method is adapted for model parameter estimation in system-level prognostic because (1) it can be applied for linear/non-linear models, (2) it can effectively handle a great number of parameters at the same time, which is the case in system-level prognostic, (3) it is an adaptable method thanks to its many extensions (Ruder, 2016), and (4) compared to Newton's method or inversion of the Hessian using conjugate gradient techniques, it is not computationally intensive, making it suitable for an online application.

In this framework, the IIM parameters are identified to minimize the mean squared error (MSE) between the inoperability estimated by the model, $\hat{q}_i$, and the in-field measured inoperability, q_i:

$$\mathcal{L}(\hat{q}_i, q_i) = \frac{1}{N}(\hat{q}_i - q_i)^2 \tag{6}$$

Algorithm 1 General algorithm for estimating the IIM parameters related to a component i

1. Set initial values of IIM parameters (a_{ij}^0, κ_i^0, θ_i^0)
2. at the $(h+1)$-th iteration step ($h \in N^+$), **while** stopping criterion not satisfied
   - Evaluate $\hat{q}_i(t) = \kappa_i^h \left[\sum_{j=1, j\neq i}^{M} a_{ij}^h q_j(t-1) + c_i(t, \theta_i^h) \right]$
   - Calculate the gradients regarding each parameter: $\frac{\partial \mathcal{L}}{\partial \kappa_i^h}, \frac{\partial \mathcal{L}}{\partial a_{ij}^h}, \frac{\partial \mathcal{L}}{\partial \theta_i^h}$
   - Update the IIM parameters:

$$\kappa_i^{h+1} = \kappa_i^h - \gamma \frac{\partial \mathcal{L}}{\partial \kappa_i^h}$$

$$a_{ij}^{h+1} = a_{ij}^h - \gamma \frac{\partial \mathcal{L}}{\partial a_{ij}^h}$$

$$\theta_i^{h+1} = \theta_i^h - \gamma \frac{\partial \mathcal{L}}{\partial \theta_i^h}$$

3. end **while**

Algorithm 1 describes how to determine all the IIM parameters, including the internal inoperability evolution of every component $c_i(t, \theta_i)$, the interdependencies matrix A and the matrix of the external influencing factors encompassed in matrix κ. Without loss of generality, let us consider that $c_i(t, \theta_i)$ is a differentiable multi-variable function of parameters θ_i that need to be estimated. In Algorithm 1, the stopping criterion can be set as a fixed number of iterations, a given value of MSE (the less is the MSE, the more is the accuracy of the model) or when an optimum is reached (a null gradient).

Depending on the prior knowledge available about the system degradation mechanisms, Algorithm 1 can be adapted easily in order to estimate only the unknown parameters.

Once the model has been formulated and its parameters determined, it will be used to estimate and predict the system health state, as explained in the next subsection.

4.2. System Health State Estimation and Prediction

To estimate the health of the system and its related uncertainty, the degradation model (i.e., IIM) and monitoring data are used in the Bayesian filtering (BF) approach. Since real systems are generally non-linear and present non-Gaussian noise, a widely-used method to obtain a sub-optimal solution for the BF problem is particle filtering (PF) (M. Orchard, 2006). In addition to the current health state estimation, this method is also used to predict the system's future health state, as described below.

4.2.1. Inoperability Uncertainty Estimation

In discrete time framework, to estimate the inoperability posterior density of the M system components at each time instant k given the observations y_k, the particle filtering (PF) is used. However, contrary to its traditional utilization, in this paper, a particle is considered as a vector representing the system components' state of health (inoperability). Thus, the weight associated with a particle represents the approximation of all the M components' inoperability probabilities at the same time. That means that each particle's weight represents the probability that the system components have particular values of inoperability contained in the particle's vector. The process of estimating the inoperability state of a system at a time k is explained in the following.

Firstly, using the IIM presented in Section 3, the prior probability density distributions PDFs of the system components inoperabilities $p(q_k|q_{k-1})$ at time k are predicted based on the ones at the previous time $k-1$:

$$p(q_k|q_{k-1}) \sim IIM(q_{k-1}) \tag{7}$$

Next, given new observations $y_{\underset{i}{k}}$ at time k for a component i, $i \in \{0, 1, ..., M\}$, the system posterior PDFs inoperabilities are updated by the particle filtering. In detail, considering a set of N particles $\{q^{(l)}\}_{l=1,...,N}$, their associated normalized weights $\{w^{(l)}\}_{l=1,...,N}$ are evaluated by the likelihood functions $p(y_{\underset{i}{k}}|q_{\underset{i}{k}})$ using the importance distribution functions $\pi(q_{\underset{i}{k}}|q_{\underset{i}{k-1}}, y_{\underset{i}{1:k}})$:

$$w_k^{(l)} \propto w_{k-1}^{(l)} \prod_{i}^{M} \frac{p(y_{\underset{i}{k}}|q_{\underset{i}{k}}^{(l)})p(q_{\underset{i}{k}}^{(l)}|q_{\underset{i}{k-1}}^{(l)})}{\pi(q_{\underset{i}{k}}|q_{\underset{i}{k-1}}, y_{\underset{i}{1:k}})} \tag{8}$$

Finally, to overcome the degeneracy problem, a resampling process is applied in each time step to replace particles having low importance weights with particles that have higher importance weights.

The posterior PDFs of the system inoperability at time k can be approximated before the resampling step by:

$$p(q_k|y_{0:k}) \approx \sum_{l=1}^{N} w_k^{(l)} \delta_{q_k}^{(l)}(q_k), \tag{9}$$

where $\delta(\cdot)$ denotes the Dirac delta function.

The estimation procedure is repeated at every instant k, $k \in \{1, 2, ..., k_p\}$, where k_p is the starting time of the prediction step presented in the next subsection.

4.3. Inoperability Uncertainty Prediction

Prognostic is a problem that goes beyond the scope of filtering problem since it involves future time horizons in which no measurements are available. Thus, the particle filtering, which is more suitable for *estimation* problems, need to be replaced by Monte Carlo simulation to perform *prediction*.

In this work, to reduce the computation requirement, we suggest to follow the procedure proposed in (Doucet, Godsill, & Andrieu, 2000) and which is based on the assumption that the particle weights are constant from time k_p to time k. According to this procedure, the predicted PDF of the inoperability of the system's components at time k (i.e., $p(q_k|y_{1:k_p})$) can be obtained by applying recursively Eq.(7) to $q_{k_p}^{(l)}$.

Once the prediction of the future system inoperability is performed, it will be used to determine the system remaining useful life (SRUL), as explained in the next subsection.

4.4. SRUL Determination

The SRUL provides information related to the time when the whole system fails (i.e., when the combined failures of individual components lead to system failure) (Rodrigues, 2018). However, the consequence of the degradation of one or more components depends on the considered architecture (e.g., parallel or series). Therefore, the SRUL must be calculated according to the system configuration.

Assuming that the system is healthy at time k_p-*th*, moment when the prediction algorithm is launched, the SRUL can be computed as follows:

$$SRUL = \tau_F - k_p, \tag{10}$$

where τ_F is the system time-of-failure ToF with:

$$\tau_F = \inf(k \in N : \text{system failure at } k) \tag{11}$$

To determine the ToF, let us denote a healthy system (with no occurrence of catastrophic failure) and a faulty system (with the occurrence of catastrophic failure) at k-*th* by H_k and F_k, respectively. Let us also consider $H_{k_p:k} = (H_{k_p}, H_{k_p} + 1, \cdots, H_k)$ as the sample space that determines all possible sequences where a system has not catastrophically failed until the time k. Then, according to the definition of the conditional probability, the failure probability without considering maintenance (i.e., the system can only fail once) at k-*th* is given by:

$$P(F_k) = P(F_k|H_{k_p:k-1})p(H_{k_p:k-1}); \forall k > k_p, \tag{12}$$

where $P(F_k|H_{k_p:k-1})$ is given by:

$$P(F_k|H_{k_p:k-1}) = \int_{R^{n_q}} p(failure|q_k)p(q_k|y_{1:k_p})d_{q_k} \tag{13}$$

The second term of Eq.(12), $p(H_{k_p:k-1})$, stands for the probability that one component is healthy from k_p-*th* until time $(k-1)$-*th*, which can be expressed as:

$$p(H_{k_p:k-1}) = \prod_{h=k_p+1}^{k-1} p(H_h|H_{k_p:h-1}) \tag{14}$$

As F_k and H_k are exclusive events, the failure event can be modeled as Bernoulli stochastic process. It follows that:

$$p(H_{k_p:k-1}) = \prod_{h=k_p+1}^{k-1} (1 - p(F_h|H_{k_p:h-1})) \tag{15}$$

The expressions presented in Eq.(12) and Eq.(15) are valid, whether for prognostic of a single component or complex systems. However, when considering a multi-components system, the way of characterizing $p(F_k|H_{k_p:k-1})$ will change according to the system configuration.

For example, in a series configuration of M components, the probability that a system will fail at time k, conditional that it is healthy at $k-1$, is a finite union of the components failure events. As only one component failure can appear at an instantaneous moment, the components failure events can be considered as incompatible. Then, the system failure probability can be written as:

$$p(F_k|H_{k_p:k-1}) = \sum_{i=1}^{M} p(F_{_{i}k}|H_{k_p:k-1}), \tag{16}$$

where $p(F_{_{i}k}|H_{k_p:k-1})$ is the probability that component i will fail at time k, conditional that the system is healthy at $k-1$.

$$p(F_k|H_{k_p:k-1}) = \sum_{i=1}^{M} \int_{q_k \in R^{n_q}} p(failure_i|q_{i^k})p(q_{_{i}k}|y_{_{i}1:k_p})d_{q_k} \tag{17}$$

For a parallel configuration, it is characterized by a parallel association of M components that are considered functioning in hot redundancy. In this structure, the failure of one or more elements does not cause the system's failure, but only when all the elements fail. Therefore, the probability that the system will fail at k, conditional that it was healthy at $k-1$, is a finite intersection of the components' failure events, which should be independent. This means that the failure of one or more components does not affect the remaining functioning components.

To provide the probability of failure of a system with a parallel configuration, we assume that the sampling time is very small. Then at most, only one component fails during the

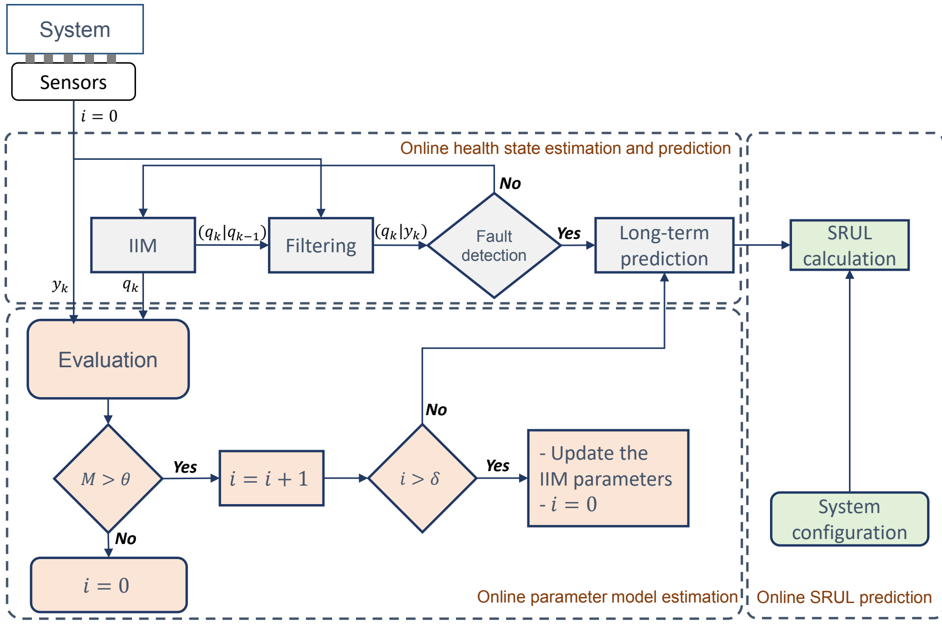


Figure 4. Methodology for online joint parameter estimation and SRUL prediction.

interval $[k-1, k]$.

$$
\begin{aligned}
p(F_k|H_{k_p:k-1}) &= \prod_{i=1}^{M} p(F_{_{ik}}|H_{k_p:k-1}) \qquad (18) \\
&= \prod_{i=1}^{M} \int_{q_k \in R^{n_q}} p(F_i|q_{i^k}) p(q_{_{ik}}|y_{_{i1:k_p}}) d_{q_k}
\end{aligned}
$$

Finally, in the case of system having a combination between series and parallel architectures (series-parallel or parallel-series) the overall system remaining useful life is assessed by decomposing the system into several series and parallel subsystems; and then each subsystem continues to be analyzed down to a single component.

5. Online Implementation of the Proposed Joint Parameter Estimation and SRUL Prediction Methodology

The main problem with the online implementation of a prognostic algorithm is its computing time (Pecht, 2009). The online implementation of the methodology proposed in this paper allows to reduce the computation time but also to address two other problems, which are:

- The online prediction of RUL/SRUL problem has been widely addressed through filtering or machine-learning methods (M. E. Orchard & Vachtsevanos, 2009). However, these methods suggest that the system degradation models are already estimated (for model-based methods) or trained (for data-driven methods) and can be used by merely updating them. Nevertheless, in practice, this information is not available. In this case, the parametric estimation of the degradation model must be done online at the same time as the system health state estimation and prediction.
- In a Bayesian approach of prognostic, the model's estimates are corrected by actual measures about the system health state without changing the parameters of the model. However, in the case of system-level prognostic, uncertainties associated with modeling can be very high. Therefore, the degradation model needs to be adaptive with regard to the monitored system.

As presented in Figure 4, the proposed methodology combined estimation of the IIM parameters and SRUL probabilistic prediction. Requiring only the trends of the component-level degradation (i.e. $c(t)$), it allows performing three principal tasks: 1) online estimation of the system health state, 2) online update of the IIM parameters, and 3) online probabilistic SRUL prediction.

In detail, the IIM, whose initial parameters were estimated offline by performing run-to-failure experiments or randomly-generated, is used at time k to predict (short-term prediction) the health state at time $k+1$ (prior estimation). At the time $k+1$, when new pre-processed degradation data acquired by sensors are available, the prior estimation is filtered to obtain the posterior one using particle filtering. If an anomaly has been detected or a threshold value for a component's inoperability has been exceeded, the posterior PDF is propagated (long-term prediction) to calculate the SRUL; otherwise, we continue filtering. After every short-term prediction, the prior health state estimation is evaluated with respect to the actual data. If there is a discrepancy, the long-term prediction is updated along with the estimated SRUL (if an anomaly is already detected). In this case, the parameter i, which represents the number of consecutive discrepancies observed, is incremented; otherwise, it is reinitialized. If several discrepancies appear consecutively (i exceeding a number δ set by the user), the gradient descent is used to update the IIM parameters.

As mentioned above, the proposed methodology requires an effective way to assess whether the difference between the measurement acquired by sensors and the predicted health state obtained by IIM is significant. In this work, the authors propose a method based on the evaluation of the uncertainty characterization. In that aim, the number of particles that fall within the accuracy range of the sensor values is determined, i.e.:

$$M = \sum_{l \in \Omega} w_k^{(l)} \; ; \tag{19}$$
$$\Omega : \{n = 1, \ldots, N | q_k \notin [y_k - \alpha\delta_{data}, y_k + \alpha\delta_{data}]\},$$

with the number of particles that should be included in the confidence interval is fixed by the user, and α is a parameter to delimits this confidence interval. Since most modern sensors are calibrated to have a Gaussian uncertainty, it is possible to use the 68–95–99.7 rule, which represents the percentage of values that lie within a band around the mean with a width of two, four, and six standard deviations, respectively. Thus, depending on the confidence that we have in the sensor measures, one percentage value can be chosen.

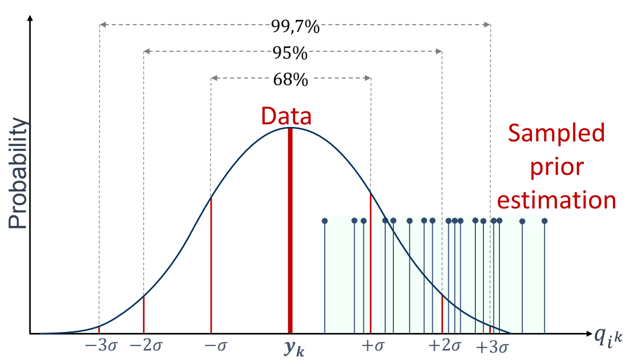


Figure 5. Evaluation of the filtering performance.

In summary, the proposed methodology allows online system state estimation and SRUL prediction, with accurate and reliable results. Indeed, the update of the IIM parameters and the long-term prediction of the component inoperability evolution is not done systematically, but only when a discrepancy is observed. This procedure prevents unnecessary computational time. Also, the parameter estimation process can be stopped when its execution time is equal to the sampling time of measurements or the loss function is close to zero. Note that the obtained values of the IIM parameters will be used as the initial values in the next iteration when a new measurement is acquired. Then, even if the optimum is not reached at a certain iteration of the algorithm, it is approached in that optimum direction. This 1) guarantees a precision of the final results in terms of parameter estimation and thus improves the accuracy of health state estimation and prediction, 2) reduces the complexity of the proposed method, as the number of iterations it takes for the gradient descent algorithm to meet its shutdown criterion decreases. Finally, the algorithm complexity decreases rapidly if parameters of the IIM are exactly priorly known to reach a quadratic complexity in case only the interactions between the components are unknown.

Another aspect to take into account in the evaluation of the prognostic performance concerns the system criticality. Indeed, depending on the consequences of the system failure, both in financial, safety, and environmental terms, the maintenance would be either optimistic or pessimistic. Over time, optimistic maintenance can be costly, while a pessimistic one can add lifespan to the system. This aspect can be easily introduced to the above-described methodology.

6. APPLICATION AND RESULTS

In this section, the proposed methodology is applied to solve the failure prognostic issue of the Tennessee Eastman Process (TEP).

6.1. System Presentation

The Tennessee Eastman Process (Downs & Vogel, 1993) is used in the literature as a realistic benchmark for process control optimization and fault diagnostics. The TEP involves five major units (working in open-loop), including a two-phase reactor, a partial condenser, a separator, a stripper, and a compressor, as shown in the schematic flow diagram and instrumentation (P&ID) of the Figure 6. The aim of this process is the synthesis of two liquid products from gaseous reactants. The process is monitored by 53 variables. In order to observe the system response, 28 faults can be injected (Bathelt, Ricker, & Jelali, 2015), which can be related to set-point changes, drifts, or random variation of variables.

As the TEP was not intended, initially, for prognostic purposes, its fundamental paradigms are changed to liken system degradation, as detailed in the next subsection.

6.2. Problem Formulation

In this case study, the authors consider failure as the interruption of the operational continuity resulting from the violation of the variables shutdown limits. Therefore, only components with shutdown constraints are considered, i.e., the reactor, the stripper, and the separator. Each of these components is monitored by a single parameter: pressure for the reactor, and liquid level for the stripper and the separator. Table 3 lists the

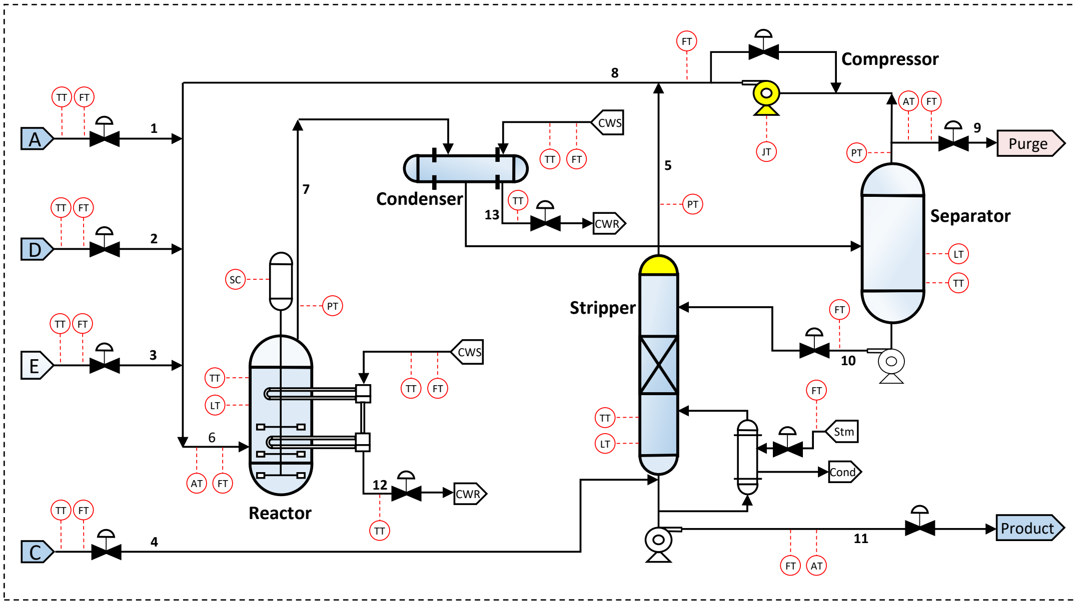


Figure 6. P&ID of Tennessee Eastman Process (Downs & Vogel, 1993).

specific operational constraints related to the system's parameter that the control system should respect.

Table 3. TEP operating constraints (Downs & Vogel, 1993).

Process variables	Operating limits		Shutdown limits	
	Low	High	Low	High
Reactor pressure (kPa)	none	2895	none	3000
Separator level (m)	3.3	9.0	1.0	12
Stripper level (m)	3.5	6.6	1.0	8.0

Two disturbances predefined in (Bathelt et al., 2015) were injected. These disturbances, occurring in the reactor and the stripper respectively, are represented as a deviation in the reactor cooling water flow and a deviation in the heat transfer of the heat exchanger of the stripper. Then, the own degradation process of the components (i.e. $c_1(t)$ and $c_2(t)$), are following the models described by equations 20 and 21, respectively.

$$c_1(t) = \alpha \cdot c_1(t-1) + \beta \tag{20}$$

$$c_2(t) = \epsilon \cdot c_2(t-1), \tag{21}$$

where α, β, and ϵ are the parameters of the two models to be estimated as well as the components of the matrix A from the monitoring data. These models are chosen based on the empirical results after investigation of numerous regression models characterizing the evolution of the monitored parameter when a single perturbation is injected in the concerned component. Regarding the matrix of influencing factors κ, its diagonal elements κ_i are equal to 1 because the data are acquired in the default production mode.

As the TEP is a continuous critical production process, to avoid financial losses and safety risks, it is not desirable to let its parameters drift until the shutdown. It is then not possible to early estimate the parameters of the degradation model and should be estimated online through the proposed methodology.

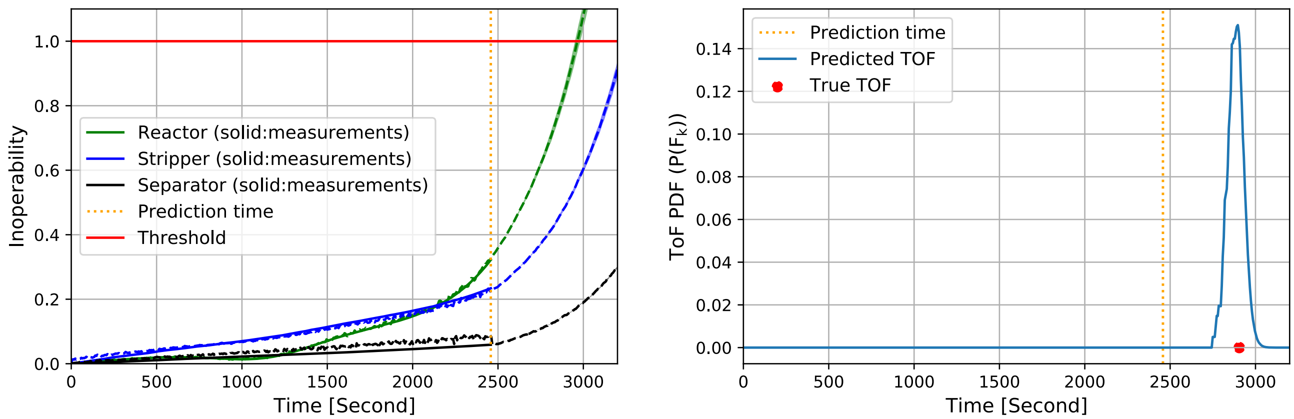
6.3. Online Parameter Estimation and SRUL Prediction of the TEP

To predict the TEP SRUL online, the methodology described in Sections 4 and 5 is utilized. This methodology's input is only the structure of the IIM, i.e., the number of critical components to monitor, failure threshold, and the trends of the component degradations.

In order to enhance the result accuracy, a digital filter is applied to the real data in order to reduce their noise. In this case, a Savitzky-Golay filter (Savitzky & Golay, 1964) is chosen because it allows increasing the precision of the data without distorting the signal trend.

In order to reduce the computation time related to the application of the proposed methodology, one must evaluate the outputs of the estimated IIM with respect to the monitoring data to investigate whether it is necessary to update the IIM. The procedure of the IIM update and the SRUL prediction is set as follows:

- When a discrepancy between the predicted value by the IIM and the monitoring data is greater than 1 σ on both sides of the mean value (i.e. $\theta = 0.01$), which represents the process measurement standard deviation, the parameter δ is incremented by 1, and a long-term prediction of the system health state is performed.
- When three successive discrepancies are detected (i.e.



(a) (b)

Figure 7. Estimated and predicted component inoperabilities (a) and ToF PDF (b) at $t_p = 2440s$.

$\delta = 3$), the IIM parameters will be updated using the GD method.

Concerning the GD-based parameter estimation method, we consider as a stopping criterion the difference of the MSE in two successive iterations less than 10^{-10}, and the learning rate γ is set to 0.005 (i.e., $\gamma = 0.005$). The initial values of the component internal degradation parameters, i.e. α, β and ϵ in equations 20 and 21, are set randomly (in order to show the robustness of the estimation method). The IIM parameters are updated throughout the TEP operation, and, at the end of the implementation, the internal degradation models of the components obtained as follows:

$$c(t) = \begin{vmatrix} c_1(t) \\ c_2(t) \\ c_3(t) \end{vmatrix} = \begin{vmatrix} 1.018 \cdot c_1(t-1) + 0.001 \\ 0.9 \cdot c_2(t-1) \\ 0 \end{vmatrix}$$

Also, the estimated interdependencies matrix A is:

$$A = \begin{bmatrix} 0 & 8 \cdot 10^{-3} & 2 \cdot 10^{-8} \\ 3 \cdot 10^{-4} & 0 & 3 \cdot 10^{-8} \\ 2 \cdot 10^{-4} & 10^{-4} & 0 \end{bmatrix}$$

One can notice that the last column elements of the estimated matrix A are smaller compared to the other matrix elements. This is due to the fact that in this simulation, the separator is not degrading by itself and thus does not significantly influence the degradation of the other components. However, its influence on the other component degradations is not null, i.e. $a_{i3} \neq 0$. In fact, the separator degrades due to the influence of the other components, and as a result, it, in turn, influences them.

Figure 7a shows the estimated and measured inoperability of the TEP units at the first prognostic time $t_p = 2440s$, which corresponds to when the reactor pressure goes out of its normal operating limit given in Table 3. One can notice that the estimation given by using the IIM (determined by GD) and the particle filtering corresponds to the actual measurements of the component inoperabilities despite the system's nonlinearity properties. Also, in Figure 7b, we can notice that the predicted ToF (equal to $2895s$) PMF is close to the true ToF (equal to $2905s$), and is slightly pessimistic. This result allows early scheduling of maintenance actions and, therefore, puts the system, its operators, and its environment in a safer situation. The evolution of the predicted SRUL is shown in Figure 8. One can notice that the predicted SRUL becomes more and more accurate over time, when more and more data are collected, and converges to the true SRUL. In this case, the SRUL corresponds to the RUL of the first failed component. Indeed, the TEP can be considered as a series-configuration one because the operability of the system depends on the operability of all its components.

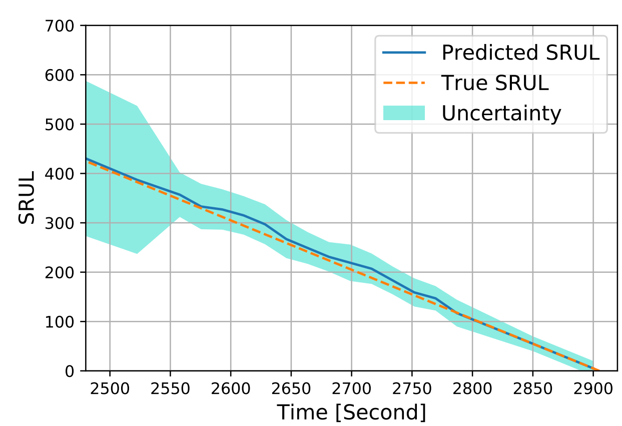


Figure 8. SRUL prediction performance with α=0.1.

By applying the proposed methodology, the IIM parameters were updated only 89 times out of a total of 494 data samples. The long-term prediction of component inoperabilities was made only 23 times, versus 82 cycles of the system after the anomaly was detected. The total computation time was 140 seconds by using an Intel core iZ 7700 and 16 Gb RAM. Knowing that the system fails after 2905 seconds of operation, it is reasonable to consider low computational resources while ensuring a good prediction of the SRUL, even though the TEP is a highly critical facility, and the resources allocated here are reasonable to deploy in reality.

6.4. Discussion

In this subsection, the proposed methodology's performance are discussed through a sensitivity analysis, and the importance of considering the components mutual interactions in the prediction results is highlighted. Finally, the advantages of using the IIM over other modeling tools are highlighted.

6.4.1. Effect of the Mutual Interaction between Components on the SRUL

In order to investigate the effect of the mutual interactions between the TEP components on its SRUL, we used the proposed methodology with IIM whose interdependence matrix (i.e. A) is null. This amounts to considering the TEP components as independent entities, which do not interact between them. In this case, the predicted SRUL at the prognostic time $t_p = 2440s$ is shown in Figure 9. One can notice that the expected value of the SRUL is equal to 780 seconds, which is later by 315 seconds than the true one (equal to 465 seconds). This prediction is too optimistic, and the difference can lead to delayed maintenance planning, causing large damages. Therefore, to avoid this bias in SRUL prediction, it is necessary to consider the mutual interactions between the components.

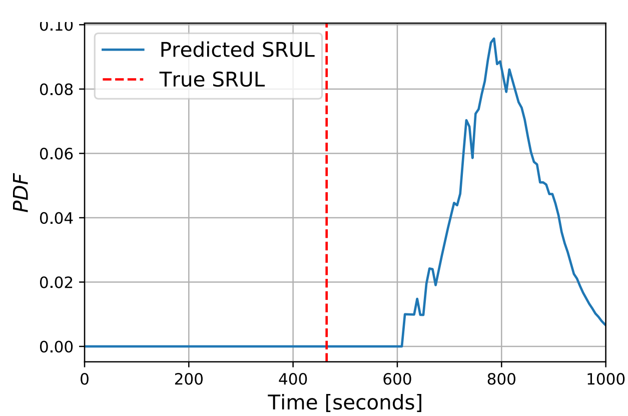


Figure 9. Predicted SRUL without taking into account interactions.

6.4.2. Sensitivity Analysis

To evaluate the sensitivity of the methodology applied here to predict the SRUL online, with respect to its parameters, i.e., θ and δ, several simulations were conducted. We firstly perform the sensitivity analysis regarding θ, i.e., the parameter used to evaluate the discrepancy between the estimate made by the IIM and the actual measurements of the system. Then, we investigate the parameter δ characterized the number of successive discrepancies needed to recompute the IIM parameters and derive the optimal value that ensures an accurate prediction of the SRUL while minimizing the computation time.

Sensitivity Analysis in regard to θ

The evaluation thresholds (confidence interval) considered for determining if there is a discrepancy between the estimate made by the IIM and the monitoring data are 1σ, 2σ, and 3σ (i.e. 1, 2 and 3 standard deviation, respectively, on both sides of the mean value), with $\delta = 3$. The result in terms of computation time, long-term prediction updates, and IIM parameter updates are shown in Table 4. One can notice that the tighter the confidence interval's width is, the more are the calculation time and prognostic updates. However, concerning the number of IIM parameter update times, they are still almost the same regardless of the value of θ considered. This can be explained by the fact that the model can be well estimated from the methodology's first iterations. However, the SRUL needs to be predicted several times because of the system's nonlinearities and noise.

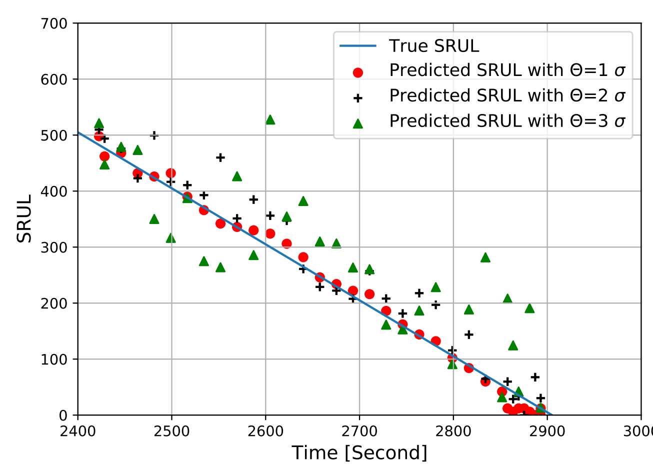


Figure 10. Predicted SRUL in function of the parameter θ.

Table 4. Calculation performances in function of the parameter θ.

Precision	$1\,\sigma$	$2\,\sigma$	$3\,\sigma$
Parameter corrections	81	81	80
Prognostic updates	91	28	27
Calculation time	140	103	88

Figure 10 shows the evolution of the predicted SRUL according to the evaluation thresholds θ, i.e. confidence interval widths. One can notice that the wider θ, the less is the accuracy of the prediction. This means that the choice of the appropriate value θ is the trade-off between two-objectives: less calculation time and better SRUL prediction accuracy.

Sensitivity Analysis in regard to δ

Figure 11 shows the variation of the computation time (represented by the green line) and the MSE (represented by the blue line) of the IIM outputs in the function of the number of successive discrepancies (i.e. δ) needed to recompute the IIM parameters. To estimate the accuracy of the GD results, an MSE is calculated at the first prognostic time, i.e., $t_p = 2440$.

One can notice that the computation time decreases when the number of discrepancies to recompute the IIM parameters is high (the inverse for the accuracy of the IIM parameters estimation). Indeed, the more the GD is performed, the closer the estimated IIM parameters to the optimal solution. Hence, by considering the reduction of the computation time and the increase of the prediction accuracy, the optimal δ value can be 4 or 5.

6.5. Advantages of using the IIM as Degradation Modeling

Regarding the modeling performance, our proposed model has shown its superiority over the existing models in prognostic literature. One can cite few notable studies that have dealt with the problem of prognosis at the system-level have used modeling tools such as state-space representation, Petri nets, fault trees, and Bayesian belief network. For example, in (Daigle et al., 2012), the state-space model was used in a distributed fashion. However, the use of the state-space model can require extensive modeling effort and expertise especially for systems having heterogeneous and interconnected components. Besides, fault trees used in (Rodrigues et al., 2014) are compelling tools, but their disadvantage is that the basic events must be independent. In (Ribot, Pencolé, & Combacau, 2008; Blancke et al., 2018), based on Petri nets, a generic online health monitoring architecture was proposed. This architecture is capable of using several prognostic methods for different components, depending on the available models. However, it does not consider the operating conditions (mission profile) during the system's utilization. For Bayesian belief network, used in (Ramasso & Gouriveau, 2014), requires a large amount of data to properly estimate the prior distributions, and calculation of the conditional probabilities can require a lot of computational resources.

In addition, the proposed IIM model addresses many of the above limitations and has several advantages. Firstly, the IIM allows modeling separately the degradation specific to each component and the degradation due to the interactions with other components. Therefore, the obtained model is generic as it can be reused for modeling other systems by only changing the interdependency matrix A. Secondly, the factor κ, which represents the influence of the operating conditions (or mission profiles), is not directly a part of the degradation model, but it is a parameter that allows modifying the evolution of the degradation. This will make it possible to determine a direct relationship between system degradation and its mission profile to minimize the degradation and maximize the SRUL. Thirdly, by normalizing the health indicators in the IIM, heterogeneous systems can be considered. Indeed, in a complex system, several components function in different ways to perform sub-tasks and achieve its primary function. Therefore, the components may have different degradation mechanisms and will be assessed with different health indicators and failure thresholds. The IIM is particularly adapted for this type of system since it proposes a single indicator of degradation: the inoperability, and a unified failure threshold (i.e. $q(t) = 1$). The last advantage of using the IIM concerns improved communication with the managers of the systems. Indeed, it is not apparent for a layperson to visualize the state of degradation of a component by referring to its health indicator (i.e., $x(t)$). To overcome this situation, it is enough to multiply the inoperability by 100 to obtain a percentage of a component's degradation, which is easily understandable.

7. CONCLUSION

In this paper, a new methodology for online system remaining useful life (SRUL) prediction is proposed. In that perspective, a unified model for the system degradation, which considers interdependencies between components, mission profile, and inner component degradations, namely the inoperability input-output model, is proposed. This methodology combines system degradation parameter determination (using gradient descent method), system health state estimation and prediction (using particle filtering), and SRUL calculation based on the system configuration. Process and data uncertainty is accounted for, while minimal input information on system degradation is required. Finally, this methodology is designed to be computationally resource-efficient while ensuring an accurate prediction of the SRUL thanks to its capacity to identify suitable moments to update the model and to predict SRUL. The applicability and the performance of the proposed methodology for real industrial systems were validated using data from the well-known Tennessee Eastman process. In detail, the obtained degradation model has proper physical meaning in relation to the system degradation mechanisms. Besides, the predicted SRUL converges to the actual value rapidly, even when considering low computation resources.

This work can raise several perspectives. First, to reduce the requirements on the knowledge available on the system, one can propose a general regression model or function-on-

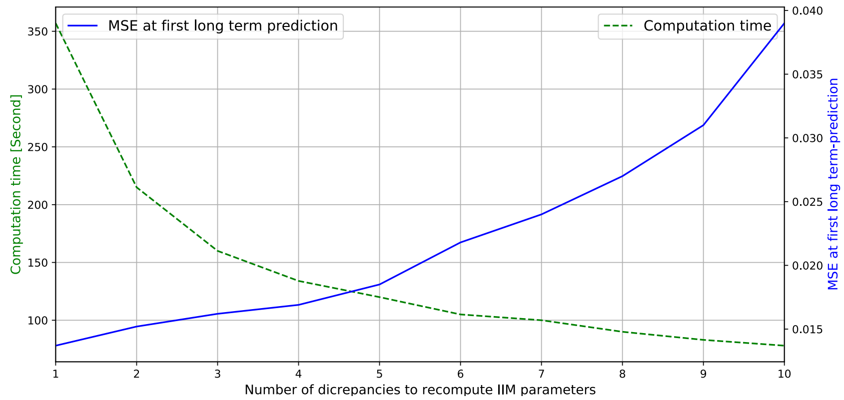


Figure 11. Computation time and accuracy of the IIM parameter estimation in function of the number of discrepancies to recompute the IIM parameters.

function model (Wang, Wang, Gupta, Rao, & Khorasgani, 2020) for the component degradations.To increase the applicability of our approach, other system architectures should be studied in the light of new paradigms for RUL calculation (Acuña-Ureta, Orchard, & Wheeler, n.d.). Besides, to consolidate the applicability and investigate the robustness of the contributions presented in this paper, other TEP modes and faults need to be studied.

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